Scalable Optimal Traffic Grooming in WDM Rings Incorporating Fast RWA Formulation

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***Abstract*—We present a scalable formulation for the traffic grooming problem in WDM ring networks. Specifically, we modify the ILP formulation to replace the constraints related to routing and wavelength assignment (RWA), typically based on a link approach, with a new set of constraints based on the maximal independent set decomposition (MISD) that we recently developed to solve optimally the RWA problem in ring networks. Our experimental study indicates that the new formulation results in an improvement of up to two orders of magnitude in running time. Consequently, it is now possible to solve the traffic grooming problem to optimality for 16-node rings in a few seconds using commodity hardware.**

I. Introduction

In the modern world, communication services delivered via the Internet touch all of society and affect all aspects of human life. The exponentially growing demand results in a great scalability problem of the Internet, thus deploying an efficient network infrustructure emerges as an important part of the answer to this problem. The global network infrustructure is built on a foundation of optical networking technologies, first deployed in the backbone and regional parts of the network but now also reaching into the access part in the form of PON architectures. With the help of wavelength division multiplexing (WDM) technology, it is possible to transmit traffic on different wavelengths within the same optical fiber simultaneously. Nowadays, the data rate of a single wavelength is in the order of 10 Gbps, while rates in the range of 40 Gbps and higher are becoming commercially available. As a result, there could be a big gap between the capacity of wavelength channels and bandwidth requirements of individual connections in wavelength routed WDM networks. Traffic grooming addresses this issue by aggregating individual traffic requests onto wavelengths so as to improve bandwidth utilization while minimizing the use of network resources.

The research on traffic grooming mainly goes to one of the two directions. In online traffic grooming, the node grooming capabilities (in terms of available electronic ports, level of wavelength conversion, and switching capacity) are assumed to be fixed and known, and the goal is to develop online algorithms for grooming and routing of connection requests that arrive in real time. Several heuristics for solving online traffic grooming problems have been developed [7], [8], [10], [17], and simulation was used to demonstrate the effectiveness of the proposed methods.

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In offline traffic grooming, it is assumed that a set of (forecast) long-term traffic demands are established, and the objective is to minimize the total network cost while satisfying all demands (e.g., as in [2], [11]), or to maximize the total revenue by satisfying as many traffic demands as possible given certain capacity (wavelength) constraints (e.g., as in [18]). Offline traffic grooming is a fundamental network design problem that has been shown to be NP-hard [4]. Such network design problems have been formulated as integer linear programs (ILPs). Basic ILP formulations of the problem are available in [6] and [18]. One of the many objective functions in the ILP studied in the literature is minizing the number of lightpaths established to carry the traffic demands [11]. Other cost functions have also been considered; for instance, the objective of the study in [5] was to minimize the electronic switching cost of grooming traffic between lightpaths at intermediate switches, while a formulation for energy-efficient traffic grooming, i.e., one that minimizes power consumption in the grooming networks, was presented in [16].

The ILP formulations face a challenging scalability issue as they are solvable only for small network topologies [16]. For larger topologies representative of realistic networks, the ILP formulation cannot be solved to optimality within reasonable amounts of time (e.g., several hours). As a result, heuristic algorithms [1], [19] have generally been used to address the offline traffic grooming problem. Other approaches tackle the problem by manipulating the ILP formulation using decomposition or column generation techniques.

One approach [9] tackles the traffic grooming problem by decomposing the original ILP into two simpler ILPs and solving them sequentially. The first ILP addresses only the traffic routing and lightpath routing subproblems and solves them first; and the second addresses the wavelength assignment problem only and takes as input the solution of the first ILP. As the problem is divided into two sequential subproblems, the decomposition is not optimal.

In [14], a multi-level decomposition method is introduced to address the multi-layered routing and multi-rate connection characteristics of traffic grooming. In this model, traffic is decomposed into four different levels, namely, alternative path, connection route, lightpath and link levels. In [12], the objective is to design a ring network that is able to satisfy any request graph with maximum degree at most δ. The cases of $\delta = 2$ and $\delta = 3$ were solved by graph decomposition.

Another approach using column generation technique were developed in [3], [13]. Given that the main difficulty in solving the problem has to do with selecting from among an exponential number of possible paths to route the traffic demands, a heuristic algorithm using column generation for a path-based formulation of the problem was developed in [3]. The key idea was to generate an optimal subset of paths efficiently. The performance of the algorithm was verified by comparing with the upper bounds generated by Lagrangian relaxation.

In this paper, we present a new traffic grooming algorithm that integrates the MISD technique we recently developed for solving the RWA algorithm in ring networks. In Section II, we present the basic but general ILP formulation for the traffic grooming problem that is the starting point of our work, and we discuss its complexity and limitations. In Section III, we integrate the MISD RWA algorithm to the formulation to traffic grooming problem. In Section IV, we present numerical results that demonstrate the effectiveness of the approach, and we conclude the paper in Section V.

II. ILP Formulation of the Traffic Grooming Problem

Consider a connected graph $G = (N, L)$, where N denotes the set of nodes and L denotes the set of directed links (arcs) in the network. We define $|N|$ and $|L|$ as the number of nodes and links, respectively. Each directed link l consists of an optical fiber that may support W distinct wavelengths. Let $T = [t_{sd}]$ denote the traffic demand matrix, where t_{sd} is a non-negative integer representing the traffic demand in terms of a unit capacity to be established from source node s to destination node d. In general, traffic demands may be asymmetric, i.e., $t_{sd} \neq t_{ds}$. We also make the assumption that $t_{ss} = 0, \forall s$. Finally, we denote C as the capacity of a single wavelength channel in terms of a unit capacity.

In this basic formulation, the entities of interest (i.e., decision variables) are link related. Hence, let us further denote the links outgoing from (incoming to, respectively) node n as L_n^+ (L_n^-, respectively).

Let us define the following set of decision variables:

- t_{ij}^{sd}: integer variable that indicates amount of traffic, in terms of unit capacity, from node s to node d carried on lightpaths from node i to node j;
- b_{ij}: integer variable that indicates number of lightpaths from node i to node j;
- b_{ij}^l: integer variable that indicates number of lightpaths from node i to node j which traverse link l; and
- $c_{ij}^{l,w}$: binary variable that indicates whether a lightpath from node i to node j uses wavelength w on link l.

In the following subsections, we will discuss the objective function and constraints.

A. Objective Function

In this paper, we are interested in minimizing the total network cost while satisfying all traffic demands. There are various minimizing objective functions, two of which receive extensive discussion, as shown below.

a). minimizing the total number of lightpaths to be established. This is a most commonly discussed objective in the literature [6], [11]. The corresponding objective function is expressed as

$$\text{minimize:} \quad \sum_{i,j} b_{ij} \tag{1}$$

b). minimizing the amount of electronically switched traffic. This objective function is used in [5]. The corresponding objective function is expressed as

$$\text{minimize:} \quad \sum_{i,j} \sum_{s,d} t_{ij}^{sd} - \sum_{s,d} t^{sd} \tag{2}$$

Here in this paper, we propose a combined objective functions, using a weight parameter, α, to join the two objectives together.

$$\text{minimize:} \ \alpha \sum_{i,j} b_{ij} + (1-\alpha) \left(\sum_{i,j} \sum_{s,d} t_{ij}^{sd} - \sum_{s,d} t^{sd} \right) \tag{3}$$

Using this objective function, one can consider minimizing the number of lightpaths and the amount of electronically switched traffic at the same time, and control the weight of each objective in the combined function by adjust the parameter α.

B. Constraints

With the notations defined before, the ILP formulation can then be expressed as:

Routing constraints:

$$\sum_{l \in L_n^+} b_{ij}^l - \sum_{l \in L_n^-} b_{ij}^l = 0, \quad n \in N \setminus \{i, j\}, i, j \in N \tag{4}$$

$$\sum_{l \in L_i^+} b_{ij}^l = b_{ij}, \quad i, j \in N \tag{5}$$

$$\sum_{l \in L_i^-} b_{ij}^l = 0, \quad i, j \in N \tag{6}$$

$$\sum_{l \in L_j^+} b_{ij}^l = 0, \quad i, j \in N \tag{7}$$

$$\sum_{l \in L_i^-} b_{ij}^l = b_{ij}, \quad i, j \in N \tag{8}$$

Wavelength constraints:

$$\sum_{w} c_{ij}^{w,l} = b_{ij}^l, \quad i, j \in N, l \in L \tag{9}$$

$$\sum_{i,j} c_{ij}^{w,l} \leq 1, \quad \forall w, \quad l \in L \tag{10}$$

$$\sum_{l \in L_n^+} c_{ij}^{w,l} = \sum_{l \in L_n^-} c_{ij}^{w,l}, \quad n \in N \setminus \{i,j\}, i,j \in N, \forall w \quad (11)$$

$$\sum_{l \in L_i^+} c_{ij}^{w,l} \leq b_{ij}, \quad i,j \in N, \forall w \quad (12)$$

$$\sum_{l \in L_i^-} c_{ij}^{w,l} = 0, \quad i,j \in N, \forall w \quad (13)$$

$$\sum_{l \in L_j^+} c_{ij}^{w,l} = 0, \quad i,j \in N, \forall w \quad (14)$$

$$\sum_{l \in L_j^-} c_{ij}^{w,l} \leq b_{ij}, \quad i,j \in N, \forall w \quad (15)$$

Grooming constraints:

$$\sum_{s,d} t_{ij}^{sd} \leq b_{ij} C, \quad i,j \in N \quad (16)$$

$$\sum_{s,d} t_{ij}^{sd} \geq (b_{ij} - 1) C, \quad i,j \in N \quad (17)$$

$$\sum_{j} t_{ij}^{sd} - \sum_{j} t_{ji}^{sd} = 0, \quad i \in N \setminus \{s,d\}, s,d \in N \quad (18)$$

$$\sum_{j} t_{ij}^{sd} = t^{sd}, \quad s,d \in N \quad (19)$$

$$\sum_{j} t_{js}^{sd} = 0, \quad s,d \in N \quad (20)$$

$$\sum_{j} t_{dj}^{sd} = 0, \quad s,d \in N \quad (21)$$

$$\sum_{j} t_{jd}^{sd} = t^{sd}, \quad s,d \in N \quad (22)$$

The routing constraints (4) to (8) find the routes for each lightpath, utilizing a multi-commodity flow equation, where each lightpath between a node pair corresponds to a single commodity. Constraint (4) ensures the number of incoming lightpaths is equal to the number of outgoing lightpaths at any intermediate node. Constraint (5) and (6) are the lightpath constraints at origin nodes of each lightpath, while constraints and (7) and (8) are the corresponding constraints for the termination nodes of each lightpath.

The wavelength assignment constraints (9) to (15) correspond to the two wavelength constraints in the problem: a). distinct wavelength constraint: (10) guarantees that each wavelength can only be used once on any link, i.e., no two lightpaths that share a common link will use the same wavelength; b). wavelength continuity constraint: multi-commodity flow equation (11) to (15) guarantee that each link on the same lightpath will be assigned the same wavelength. Finally, constraint (9) ensures that each lightpath will only use one wavelength.

The grooming assignment constraints (16) to (22) are for grooming traffic into lightpaths over the virtual topology. Constraint (16) is the channel capacity constraint that counts the number of lightpaths need to be established between each node pair, while (17) ensures that only enough number of lightpaths are to be established. Constraints (18) to (22) are multi-commodity flow equations that find the traffic route(s) for each connection request.

The scalability of an ILP formulation depends directly on its size, which, in turn, is determined by the number of variables and constraints. The above formulation consists of $N^2(N-1)^2$ integer variables $\{t_{ij}^{sd}\}$, $N(N-1)$ variables $\{b_{ij}\}$, $N(N-1)|L|$ variables $\{b_{ij}^{l}\}$, and $N(N-1)|L|W$ binary variables $\{c_{ij}^{l,w}\}$, for a total of $O(N^2|L|W)$ variables. For the number of constraints, there are $N(N-1)(N+2)$ routing constraints, $O(N(N-1)(N+2)W)$ wavelength constraints, and $N(N-1)(N+4)$ grooming constraints. Hence there are in total $O(N^3W)$ constraints in the formulation.

This existing ILP formulation has two main limitations: (1) its size increases rapidly with the size of the network; and (2) they have a symmetry problem in that multiple solutions with the same objective value can be obtained by simply changing the order of wavelengths. Since the ILP solver has to evaluate all $W!$ distinct optimal solutions, the running time can be unnecessarily long. Hence, these formulations do not scale to networks with 100 or more wavelengths per link that can be realized with current technology.

In the following section, we will integrate our recently developed MISD approach, which response well to the two big challenges above.

III. Routing and Wavelength Assignment in Traffic Grooming

The routing and wavelength assignment algorithm takes a large portion of efforts in the optimization procedure of CPLEX. As stated earlier, itself is a NP-hard problem. Especially when it is combined with the grooming assignment, it adds up the complexity to the whole problem dramatically. In our recent research [15], we discussed in depth the performance of existing approaches for solving the RWA problem and developed an exact decomposition approach for an ILP formulation based on maximal independent sets that makes it possible to obtain optimal solutions to the RWA problem for maximum size (i.e., 16-node) SONET rings in only a few seconds using commodity CPUs. This new, fast technique, maximal independent set decomposition (MISD) approach, achieves several orders of magnitude decrease in running time compared to existing formulations (link-based and path-based).

Looking back at the conventional ILP formulation to traffic grooming problem, we find that it utilizes the most complicated RWA ILP formulation, i.e., the link-based formulation. Altough it ensures overall optimality to arbitary network topologies, the link-based formulation results in an enormous complexity to the approach. Since the MISD approach is equivalent to link-based formulation in ring networks, we incorporate it into the traffic grooming formulation. Doing so, as we will show shortly, greatly increase the efficiency of the traffic grooming solution in rings *without any sacrifice in optimality*.

Before we introduce the equation substitution, a quick overview of MISD approach is warranted. MISD is a decomposition approach based on a maximal-independent-set-based formulation of the RWA problem. Taking MISD-2 in [15] as an example, this approach decomposes the network into two paritions that have no links in common. It can perfectly fit in the ILP for traffic grooming problem by substituting Equation (4)-(15) with (23)-(25). For more details for the approach and understanding the equations below, the reader may refer to [15] .

$$\sum_{k=cw,ccw} b_{ij,k} = b_{ij} \qquad i,j \in N \tag{23}$$

$$b_{ij,k} \leq \sum_{m \in \mathcal{M}^k} v_m^k X_{ij,k}^m \qquad i,j \in N, k = cw, ccw \tag{24}$$

$$\sum_{m \in \mathcal{M}^k} v_m^k \leq W \qquad k = cw, ccw \tag{25}$$

Moreover, MISD-4 is an advanced version of MISD-2, which decomposes the network into 4 partitions instead of 2 partitions in MISD-2. Again, the formulation can be found in [15] and can be substituted in the traffic grooming ILPs in a similar manner. We will use both MISD-2 and MISD-4 in the new formulation and have a performance evaluation in the following section.

IV. Numerical Results

In this section, we present the results of an experimental study we conducted to investigate the performance of the traffic grooming with MISD, compared to the original approach. All results were obtained by running the IBM Ilog CPLEX 12 optimization tool on a cluster of identical compute nodes with dual Woodcrest Xeon CPU at 2.33GHz with 1333MHz memory bus, 4GB of memory and 4MB L2 cache.

Our study involves a large set of problem instances on ring networks with size up to 16 nodes (the largest possible size of a SONET Ring). We set the capacity of each wavelength to $C = 16$. For each network size, we consider several problem instances. For each problem instance, the traffic demand matrix $T = [t_{sd}]$ is generated by drawing the (integer) traffic demands uniformly at random in the interval $[0, T_{max}]$. Each data point in the figures we present in this section represents the average of 10 random problem instances for the given topology and value of parameters T_{max}. Note that, as we mentioned earlier, incorporating MISD within the traffic grooming formulation does not affect optimality, i.e., the solution obtained has the same objective value as that obtained via the original grooming formulation (although the exact structure of the solution may be different). Therefore, we focus only on the running time performance of the new formulations.

In Figures 1, 2 and 3 we plot the running times of the original traffic grooming ILP, the ILP with MISD-2 RWA, and the ILP with MISD-4 RWA, as a function of the size of the ring network. We set the value of parameter $\alpha = 0.2$ in Figure 1 to represent considering minimizing power electronically switched traffic as the main objective, $\alpha = 0.8$ in Figure 3 as to represent considering minimizing the number of lightpaths established as the main objective, and $\alpha = 0.5$ to represent considering the two objectives equally weighted.

As we can see, in all three situations, the ILPs that incorprate the MISD techniques achieve around two orders of magnitude decrease in running time. Particularly, the ILP with the MISD-4 technique performs well in large networks, and it can solve the largest SONET ring (16-node) network in about two CPU seconds for all three values of parameter α we considered in this study. As a result, the new formulation makes it possible to solve network sizes of practical size fast enough to allow network designers and operators to perform extensive "what-if" analysis so as to investigate large numbers of scenarios regarding forecast demands.

Figure 4 compares the running times of the original ILP and the ILPs incorporating MISD as a function of T_{max}, on a 16-node ring network; note that a higher value of T_{max} corresponds to a higher traffic load for the whole network. As we can see in the figure, again, the new approaches achieve two orders of magnitude decrease in running time, and this result is consistent across the traffic loads shown.

Finally, Figure 5 compares the running time of the three ILP formulations as a function of the available number of wavelengths. In this figure, we select again the 16-node ring network and set $T_{max} = 20$. Similar to the result shown earlier, the new approaches achieve two orders of magnitude decrease in running time, consistently across the range of wavelengths we investigated.

From a practical perspective, the new ILP formulation incorporating MISD-4 make it possible to solve traffic grooming optimally for maximum-size (16-node) SONET rings with the combined objective in about two seconds; importantly, the running time is not sensitive to the traffic load or the number of wavelength available.

V. Concluding Remarks

We have presented a new approach to solve the traffic grooming problem with MIS decomposition in ring networks optimally. The new formulation scales well and enables network designers and operators to carry out extensive "what-if" analysis.

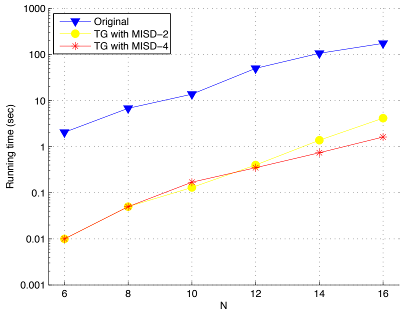

Fig. 1. CPU time comparison vs. N, $\alpha = 0.2$

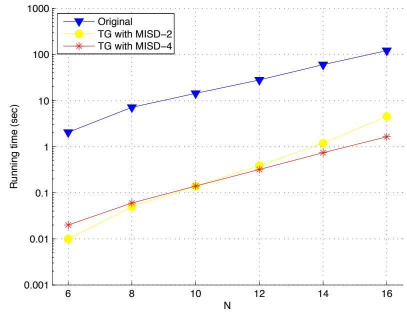

Fig. 2. CPU time comparison vs. N, $\alpha = 0.5$

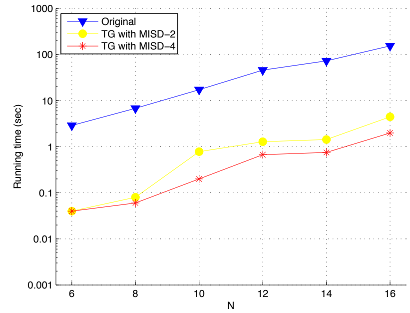

Fig. 3. CPU time comparison vs. N, $\alpha = 0.8$

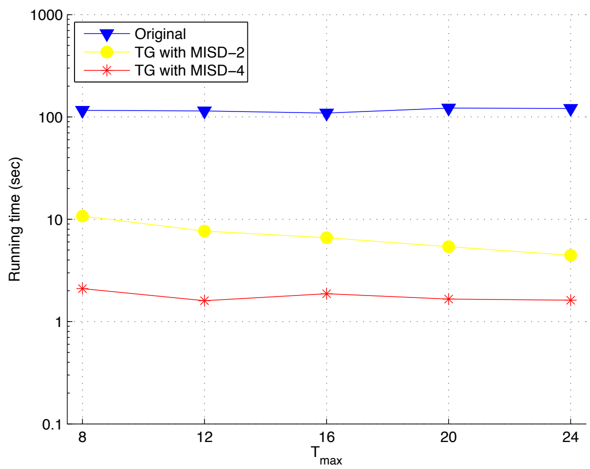

Fig. 4. CPU time comparison vs. T_{max} at $N = 16$

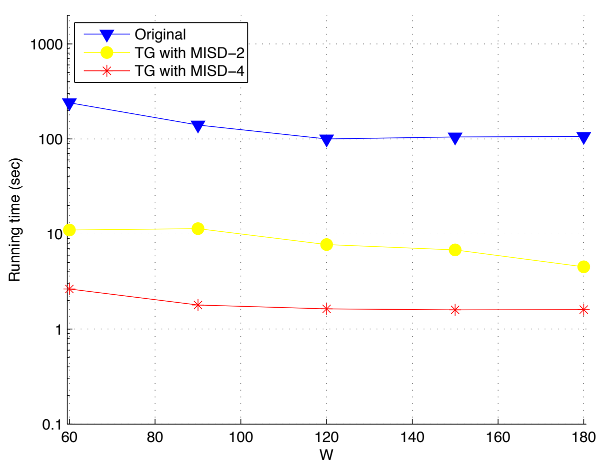

Fig. 5. CPU time comparison vs. W at $N = 16$

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