# Analysis and Optimization of Transmission Schedules for Single-Hop WDM Networks

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## Abstract

We consider single-hop lightwave networks with stations interconnected using Wavelength Division Multiplexing. The stations are equipped with tunable transmitters and/or receivers. Coordination between the transmitting and receiving stations is achieved by assuming synchronous control and a predefined, frequency-time oriented schedule which specifies the slots and the wavelengths on which communication between any two pairs of stations is allowed to take place. We define and analyze, in terms of throughput, all possible types of schedules in the situation where the number of available wavelengths is equal to the number of stations. Our results are valid for the general case, i.e., non-uniform traffic. We then consider the optimization of schedules given the traffic requirements and present optimization heuristics that give near-optimal results.

## 1 Introduction

Wavelength Division Multiplexing (WDM) is emerging as a promising technology for the next generation of multiuser high speed communication networks. WDM introduces transmission concurrency by dividing the low-loss wavelength spectrum of the optical fiber into a number of independent channels. As a result, WDM networks have the potential of delivering an aggregate throughput that can grow with the number of wavelengths deployed, and can be in the order of Tbps.

In single-hop WDM networks packet transmissions are possible only when a direct communication path is established between the source and the destination and tunable lasers and/or filters with a large tuning range × tuning speed product are required in order to fully utilize the capabilities of WDM [2]. Research in this area has focused on the problem of allocating the bandwidth among the network stations. Numerous protocols have been devised (see [11] for an overview). Protocols based on pretransmission coordination [10, 3, 4, 8] employ one or more shared control channel for the arbitration of the transmission requests. Protocols with no pretransmission coordination can employ either random access schemes [7] or a predetermined, frequency-time assignment of the optical bandwidth [5].

The frequency-time assignment technique is the extension of time division multiplexing over a multi-channel environment. A schedule specifies the slots within each frame and the channel on which packet transmissions are permitted between any source-destination pair. Our work deals with analysis of all types of schedules when the number of available wavelengths is equal to the number of stations and the traffic could be non-uniform. Furthermore, we consider the problem of schedule optimization, given the traffic requirements. Although this is a hard problem, we are able to derive heuristics that give near-optimal results.

Similar optimization problems, although in a different context, are addressed in [9, 1]. In [6, 5, 13] a number of schedules is studied and models are developed to analyze their performance. These works, in contrast with ours, do not deal with schedule optimization, and the analysis is restricted to uniform traffic.

This paper is organized as follows. In Section 2 we describe our system model and in Section 3 we obtain expressions for the throughput of all types of schedules. Section 4 investigates the problem of obtaining an optimal schedule and Section 5 presents a heuristic which yields very good results. In Section 6 a more general heuristic is developed. Section 7 presents some numerical examples and Section 8 contains some concluding remarks.

## 2 System Model

We consider a network of $N$ stations, interconnected through an optical broadcast medium that can support $N$ wavelengths, $\lambda_1, \lambda_2, \ldots, \lambda_N$. Each station is equipped with one receiver and one transmitter. The properties of the network depend on whether the transmitters, the receivers, or both are tunable. Following the terminology in [11] we refer to the three resulting types of systems as TT-FR, FT-TR and TT-TR, respectively. If the receivers (transmitters) are fixed, wavelength $\lambda_i$ is assigned to the receiver (transmitter) of station $i$. The tunable transmitters (receivers), on the other hand, consist of lasers (filters) tunable over a wide range of wavelengths which includes all $\lambda_i$.

The network operates in a slotted mode, with a slot time equal to the packet transmission time plus the tuning time, and all stations are synchronized to the slot boundaries. Let $\sigma_i$ be the probability that a new packet arrives at station $i$ during a slot time. Let $p_{ij}$ be the probability that a packet arriving at station $i$ is destined to station $j$, and $\sum_j p_{ij} = 1$. Each station has $N-1$ buffers, one buffer for storing a single packet to each destination. If a packet for station $j$ arrives at station $i$ while a packet for $j$ is already stored in $i$'s buffer, the new packet is lost. This is an extension of the model found in [9].

We assume single hop communication. Coordination between the transmitting and receiving stations is achieved by using a predefined, frequency-time oriented schedule that works as follows. Time slots are grouped in frames of $M$ slots. Within a frame, a number of $a_{ij}$ slots is assigned for packet transmissions between the source-destination pair $(i,j)$. A schedule indicates, for all $i$ and $j$, which slots during a frame can be used for transmissions from $i$ to $j$, and can be described by the variables $\delta_{ij}^{(t)}, t = 1,\dots,M$, defined as ($a_{ij} = \sum_{t=1}^{M} \delta_{ij}^{(t)}$ )

$$\delta_{ij}^{(t)} = \begin{cases} 1, & \text{if } i \text{ can transmit to } j \text{ in slot } t \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

We also define $w_{ij}^{(t)} \in \{\lambda_1,\dots,\lambda_N\}$ as the wavelength on which $i$ will transmit a packet to $j$ in slot $t$. This is useful only for TT-TR systems since for a TT-FR (FT-TR) system we have that $w_{ij}^{(t)} = \lambda_j \ \forall \ i,t$ ($w_{ij}^{(t)} = \lambda_i \ \forall \ j,t$).

### 2.1 Transmission Modes

We define the *transmitting set*, $I_i^{(t)}, t = 1,\dots,M$, of station $i$ in slot $t$ as the set of stations that $i$ is permitted to transmit to in slot $t$. The *receiving set*, $J_j^{(t)}, t = 1,\dots,M$, of station $j$ in slot $t$ is the set of stations that have permission to transmit to $j$. In terms of $\delta_{ij}^{(t)}$ we can write

$$I_i^{(t)} = \{j \mid \delta_{ij}^{(t)} = 1\} \ \forall \ i,t \ \ , \ \ J_j^{(t)} = \{i \mid \delta_{ij}^{(t)} = 1\} \ \forall \ j,t \quad (2)$$

The following will also be true at all times

$$j \in I_i^{(t)} \iff i \in J_j^{(t)} \quad \forall \ i,j,t \quad (3)$$

Given a schedule $S$ we distinguish four different transmission modes with respect to the possible values that the cardinalities of sets $I_i^{(t)}$ and $J_j^{(t)}$ can take.

**Many-to-one mode:** The transmission mode in a slot $t$ is called many-to-one if no station is permitted to transmit to more than one destination in the slot. In terms of transmitting sets, $\mid I_i^{(t)} \mid = \sum_{j=1}^{N} \delta_{ij}^{(t)} = 1 \ \forall \ i$. There is no restriction on the cardinality of the receiving sets, allowing for the possibility that a number of sources are permitted to transmit to the same destination in slot $t$, thus the name many-to-one. However, the previous condition and (3) imply that $J_j^{(t)} \bigcap J_l^{(t)} = \emptyset \ \ \forall \ j \neq l$.

**One-to-many mode:** The one-to-many transmission mode is such that at most one source is permitted to transmit to a certain destination, or $\mid J_j^{(t)} \mid = \sum_{i=1}^{N} \delta_{ij}^{(t)} = 1 \ \ \forall \ j$. The transmitting sets can now take any values as long as $I_i^{(t)} \bigcap I_k^{(t)} = \emptyset \ \ \forall \ i \neq k$. If $\mid I_i^{(t)} \mid > 1$, a single station $i$ is permitted to transmit to a number of destinations during slot $t$, although at most one transmission is possible (more on this later), hence the term one-to-many.

**One-to-one mode:** This is a special case of the above two transmission modes, and is such that $\mid I_i^{(t)} \mid = \mid J_j^{(t)} \mid = 1 \ \forall \ i,j$. Transmissions between pairs $(i,j)$ and $(k,l)$ are assigned in slot $t$ if, and only if, $i \neq k$ and $j \neq l$. A station can transmit to, and receive from, only one other station.

**Many-to-many mode:** This is the most general transmission mode, as it allows $I_i^{(t)}$ and $J_j^{(t)}$ to be any sets.

### 2.2 Types of Policies

Although the transmitting set of a source may contain more than one destination, only one packet transmission is possible during a slot. The source, therefore, has the freedom to select the destination to which it will transmit a packet. Similarly, if more than one station belongs to the receiving set of a station *and* the receivers are tunable, the receiving station may select one of many possible sources to which it can tune. The criteria that the transmitting and receiving stations use to select one station in their transmitting and receiving sets respectively, define a policy. Two policies that are simple, very fast to execute and do not require coordination prior to packet transmission are now described.

**Random Policy (RP):** At the beginning of each slot $t$ for which $\mid I_i^{(t)} \mid > 1$ ($\mid J_j^{(t)} \mid > 1$ and the receivers are tunable), $i$ ($j$) selects randomly one of the stations in $I_i^{(t)}$ ($J_j^{(t)}$), say $l$ ($k$). $i$ will transmit a packet, if it has one for the selected station on wavelength $w_{il}^{(t)}$. $j$ will tune to wavelength $w_{kj}^{(t)}$ and will wait for a transmission, if any, from $k$.

**Round-Robin Policy(RRP):** All stations in $I_i^{(t)}$ ($J_j^{(t)}$) are ordered. $i$ ($j$) considers for transmission (reception) in slot $t$ only one station per frame. By remembering which station it considered during the previous frame, $i$ ($j$) considers all stations in $I_i^{(t)}$ ($J_j^{(t)}$) in a cyclic order.

## 3 Throughput Analysis

A schedule can be characterized by three parameters: (a) the type of system under consideration (TT-FR, FT-TR, or TT-TR), (b) the most general transmission mode in any of its slots, and (c) the policies used by the transmitters and receivers. The analysis will be different depending on whether collisions, destination conflicts or both are possible. This is illustrated in Figures 2 and 3 where we show how the various transmission modes differ when implemented in a TT-FR or FT-TR system, respectively. Finally, different policies will result in different system behavior.

We now proceed to derive expressions for the throughput of the different types of schedules of a TT-FR system. Expressions for the FT-TR and TT-TR systems can be found in [12]. The throughput of a schedule is given by $T = \sum_{i=1}^{N} \sum_{j=1}^{N} T_{ij}$ where $T_{ij}$ is the throughput of the source-destination pair $(i,j)$, i.e., the number of successful packet transmissions per slot between $i$ and $j$. When appropriate, the throughput will be given as a function of the policy used.

### 3.1 TT-FR Systems

#### 3.1.1 Many-To-One Schedules

As can be seen from Figure 2, a collision will occur if two or more transmitters try to transmit to the same station in a slot. Let $d_{ij}^{(t)}, t = 1,\dots,M$ be the distance (in slots) between the beginning of the last slot before $t$ that $i$ was allowed to transmit to $j$, and the beginning of $t$ (see Figure 4(a)). If $t_1$ was this last slot, then $d_{ij}^{(t)} = t - t_1$, and $\sum_{t,\delta_{ij}^{(t)}=1} d_{ij}^{(t)} = M$.

Since at most one packet with destination $j$ can be in the buffer of station $i$ at any time instant, for a many-to-one schedule we have

$$T_{ij} = \frac{1}{M} \sum_{\substack{t \\ \delta_{ij}^{(t)}=1}} \left[1 - (1 - \sigma_i p_{ij})^{d_{ij}^{(t)}}\right] \prod_{\substack{n \in J_j^{(t)} \\ n \neq i}} (1 - \sigma_n p_{nj})^{d_{nj}^{(t)}} \qquad (4)$$

Expression (4) is derived by noting that a packet will be successfully transmitted from $i$ to $j$ in slot $t$ if: (a) $i$ can transmit to $j$ in that slot, (b) $i$ has a packet for $j$ at the beginning of the slot, and (c) if another station $n$ is also allowed to transmit to $j$ in $t$, then $n$ does not have a packet for $j$ at the beginning of the slot.

### 3.1.2 One-To-One Schedules

An expression for the throughput of one-to-one schedules can be derived directly from (4) by noting that the product in the right hand side reduces to 1. However, for one-to-one schedules we do not need to define the distances $d_{ij}^{(t)}$ for all slots, but only for the $a_{ij}$ slots allocated for transmissions from $i$ to $j$, as in Figure 4(b). Let $d_{ij}^{(k)}, k = 1, \ldots, a_{ij}$, be the distance between the $k$-th slot allocated to pair $(i,j)$ in a frame, and the next such slot (in the same or next frame). Obviously, $\sum_k d_{ij}^{(k)} = M$, and we get

$$T_{ij} \;=\; \frac{1}{M}\sum_{k=1}^{a_{ij}} 1 - (1 - \sigma_i p_{ij})^{d_{ij}^{(k)}} \qquad (5)$$

### 3.1.3 One-To-Many Schedules With RRP

By definition, no collisions are possible in one-to-many schedules. Consider now the pair $(i,j)$ and let $z(i,t,f) \in I_i^{(t)}$ be the decision of the round-robin policy (the station $i$ selects to transmit to) in slot $t$ of frame $f$. Define $F_{ij} = \prod_{t, j \in I_i^{(t)}} \mid I_i^{(t)} \mid$. It is straightforward to show that the decisions of the round-robin policy for the slots that $i$ is allowed to transmit to $j$ repeat after $F_{ij}$ frames, or $z(i,t,f) = z(i,t,f+F_{ij})\ \forall\ f$. We can now restrict our attention to a window of $F_{ij}$ consecutive frames. $i$ selects to transmit to $j$ only once every $\mid I_i^{(t)} \mid$ frames within a window, for a total of $l_{ij} = \sum_{t, j \in I_i^{(t)}} \frac{F_{ij}}{|I_i^{(t)}|}$ times. Given the decisions $z(i,t,f)$, the distances between two successive slots that $i$ selects to transmit to $j$ are known. Let $d_{ij}^{(k)}, k = 1, \ldots, l_{ij}$ be the extension of the distances defined in Figure 4 (b), over a window of frames. Then, for the pair $(i,j)$ we obtain

$$T_{ij}(\text{RRP}) = \frac{1}{M\ F_{ij}} \sum_{k=1}^{l_{ij}} 1 - (1 - \sigma_i p_{ij})^{d_{ij}^{(k)}} \qquad (6)$$

### 3.1.4 One-To-Many Schedules With RP

We observe the system at the instants just before the beginning of a slot $t$ such that $j$ is in the transmitting set of $i$. Let $t$ be the $l$-th such slot, $l = 1, 2, 3, \ldots$. We define $q_{ij}^{(l)}$ as the probability that $i$ has a packet for $j$ at the beginning of slot $t$, and $\mathcal{I}_i^{(l)}$ as the transmitting set of $i$ in slot $t$ (this is similar to the definition of $d_{ij}^{(k)}$ in Figure 4 (b), but is not restricted to one frame). Also, let $d_{ij}^{(l)}, l = 1, 2, \ldots$ be an extension of $d_{ij}^{(k)}$ over all frames. $q_{ij}^{(l)}$ will be equal to 1 if $i$ had a packet for $j$ at the beginning of the $(l-1)$-th slot and $j$ was not selected by the random policy at that slot. Otherwise, $q_{ij}^{(l)}$ is equal to the probability that a packet for $j$ arrived at $i$ during the $d_{ij}^{(l-1)}$ slots between the $(l-1)$-th and the $l$-th slots. It is important to notice that for $k = 1, \ldots, a_{ij}$ and $l = 1, 2, \ldots$

$$\mathcal{I}_{ij}^{(la_i+k)} \;=\; \mathcal{I}_i^{(k)} \;,\; d_{ij}^{(la_{ij}+k)} \;=\; d_{ij}^{(k)} \qquad (7)$$

The probability that $j$ is not selected by the random policy in the $l$-th slot is just $1 - (1/ \mid \mathcal{I}_i^{(l)} \mid)$. We can now express $q_{ij}^{(l)}, l = 2, 3, \ldots$ as

$$q_{ij}^{(l)} \;=\; q_{ij}^{(l-1)}\left(1 - \frac{1}{\mid \mathcal{I}_i^{(l-1)} \mid}\right)$$

$$+\left[1 - q_{ij}^{(l-1)}\left(1 - \frac{1}{\mid \mathcal{I}_i^{(l-1)} \mid}\right)\right]\left[1 - (1 - \sigma_i p_{ij})^{d_{ij}^{(l-1)}}\right] \qquad (8)$$

$$q_{ij}^{(1)} \;=\; 0 \qquad (9)$$

The initial condition (9) is obtained by assuming that the frame starts at a slot in which $i$ can transmit to $j$. The recursive equation (8) and the initial condition (9) describe the dynamics of the system. If this system has a steady state, these quantities should converge, as $l \to \infty$, to time-independent quantities such that

$$\lim_{l \to \infty} q_{ij}^{(l+a_{ij})} \;=\; \lim_{l \to \infty} q_{ij}^{(l)} \qquad (10)$$

We now investigate the conditions under which the system has a steady state. After some algebraic manipulation of (8), we get

$$q_{ij}^{(l)} \;=\; A_{l-1} q_{ij}^{(l-1)} \;+\; B_{l-1}, \quad l = 2, 3, 4, \ldots \qquad (11)$$

$$A_l = \left(1 - \frac{1}{\mid \mathcal{I}_i^{(l)} \mid}\right)(1 - B_l), B_l = 1 - (1 - \sigma_i p_{ij})^{d_{ij}^{(l)}} \qquad (12)$$

Because of (7) we have for $k = 1, \ldots, a_{ij}, l = 1, 2, \ldots$

$$A_{la_{ij}+k} \;=\; A_k \;,\; B_{la_{ij}+k} \;=\; B_k \qquad (13)$$

It is easy to show using induction that

$$q_{ij}^{(la_{ij}+1)} = q_{ij}^{(a_{ij}+1)} \sum_{n=0}^{l-1} A^n, A = \prod_{k=1}^{a_{ij}} A_k,\ l = 1, 2, \ldots \qquad (14)$$

The sum in (14) will always converge as $l \to \infty$ since $0 \le A < 1$ (see (12)). Therefore, the system always has a steady state. Define

$$r_{ij}^{(k)} \;=\; \lim_{l \to \infty} q_{ij}^{(la_{ij}+k)} \quad k = 1, \ldots, a_{ij} \qquad (15)$$

$r_{ij}^{(k)}$ is the limiting probability that $i$ has a packet for $j$ in the $k$-th slot within a frame that $j$ is in the transmitting set of $i$. From (14) we get

$$r_{ij}^{(1)} = \frac{1}{1-A}\, q_{ij}^{(a_{ij}+1)} = \frac{1}{1-A} \sum_{m=1}^{a_{ij}} B_m \prod_{n=m+1}^{a_{ij}} A_n \quad (16)$$

We can now obtain $r_{ij}^{(k)}$ using (16) and the following recursion

$$r_{ij}^{(k)} \;=\; A_{k-1} r_{ij}^{(k-1)} \;+\; B_{k-1} \quad k = 2, \ldots, a_{ij} \quad (17)$$

The probability that there will be a successful transmission from $i$ to $j$ in the $k$-th slot in a frame, is equal to the product of the probability that $i$ has a packet for $j$ at the beginning of the slot times the probability that the random policy selects $j$ for transmission. As $l \to \infty$,

$$T_{ij}(RP) \;=\; \frac{1}{M} \sum_{k=1}^{a_{ij}} \frac{r_{ij}^{(k)}}{\mid \mathcal{I}_i^{(k)} \mid} \quad (18)$$

In the special case of a one-to-one schedule, $\mid \mathcal{I}_i^{(k)} \mid = 1$ and $A_k = 0$ for $k = 1, \ldots, a_{ij}$. Then, $r_{ij}^{(k)} = B_{k-1}$ and expression (18) reduces to (5).

#### 3.1.5 Many-To-Many Schedules With RP

We define $q_{ij}^{(l)}$, $\mathcal{I}_i^{(l)}$ and $r_{ij}^{(k)}$ as for the one-to-many schedules. We also define $\mathcal{J}_j^{(l)}$ in a way analogous to the definition of $\mathcal{I}_i^{(l)}$. $r_{ij}^{(k)}$ can be obtained by using (16) and (17). However, two or more stations in the receiving set of $j$ may select $j$ for transmission in a given slot, introducing the possibility for collisions (Figure 2). The expression for the throughput then becomes:

$$T_{ij}(RP) = \frac{1}{M} \sum_{k=1}^{a_{ij}} \frac{r_{ij}^{(k)}}{\mid \mathcal{I}_i^{(k)} \mid} \prod_{\substack{n \in \mathcal{J}_j^{(k)} \\ n \neq i}} \left[ 1 - \frac{r_{nj}^{(k)}}{\mid \mathcal{I}_n^{(k)} \mid} \right] \quad (19)$$

## 4 Schedule Optimization

We now turn our attention to the problem of obtaining schedules that maximize the system throughput. The problem can be stated concisely as: *Given traffic parameters $\sigma_i$ and $p_{ij}, i, j = 1, \ldots, N$, and given the tunability characteristics of the system, find the optimum schedule.* To ensure fairness, all schedules should have the property that at least one slot is assigned for transmission between a pair of stations $(i, j)$ if the probability of traffic originating at $i$ and terminating at $j$ is nonzero, or,

$$\forall\, i, j \quad \text{if} \;\; \sigma_i p_{ij} > 0 \;\; \text{then} \;\; a_{ij} \geq 1 \quad (20)$$

Let $Q_{ij} = 1 - (1 - \sigma_i p_{ij})^M$ be the probability that a packet with destination $j$ will arrive at station $i$ during a number of slots equal to the frame length, $M$. When $Q_{ij}$ is close to one for all source-destination pairs $(i, j)$, one-to-one schedules are favored over other types of schedules, as no packet transmissions are wasted due to collisions or destination conflicts. But in one-to-one schedules, slots are assigned for the exclusive use of a certain source-destination pair. Therefore, if the above condition is not true, some slots may be unused for most of the time. Our approach then, is to first consider determining an optimal one-to-one schedule. Next we consider how we may obtain other types of schedules with good performance in situations where $Q_{ij}$ is very low for some pairs of stations.

## 5 Optimizing One-To-One Schedules

Our goal is to determine a one-to-one schedule $S$ such that the overall throughput is maximized. This was addressed in [9] for a single channel system. Expression (5) is valid for all three types of systems. However, it is easy to show that an algorithm that takes into account that both transmitters and receivers are tunable cannot produce one-to-one schedules with better throughput than if only, say, transmitters are tunable. Thus, in the following discussion we only consider TT-FR and FT-TR systems, in which transmissions to or from a certain station respectively, take place on the same channel.

The optimization problem can be formulated as

$$P_1 : \max_{a_{ij}, d_{ij}^{(k)}, M} \; T = \frac{1}{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{a_{ij}} 1 - (1 - \sigma_i p_{ij})^{d_{ij}^{(k)}}$$

$$\text{subject to} \quad \sum_{k=1}^{a_{ij}} d_{ij}^{(k)} \;=\; M \quad \forall\, i, j \quad (21)$$

$$\sum_{i=1}^{N} a_{ij} = M \;\forall\, j \;, \; \sum_{j=1}^{N} a_{ij} = M \;\forall\, i \quad (22)$$

$$\sum_{i=1}^{N} \delta_{ij}^{(t)} = 1 \;\; \forall\, j, t \;, \; \sum_{j=1}^{N} \delta_{ij}^{(t)} = 1 \;\; \forall\, i, t \quad (23)$$

$$\delta_{ij}^{(t)} = 0, 1 \;\; \forall\, i, j, t \;,\; M, d_{ij}^{(k)}, a_{ij} \text{ integer } \;\forall\, i, j, k \quad (24)$$

Constraints (22) specify that each station receives and transmits in exactly $M$ slots, while constraints (23) guarantee that the resulting schedule is a one-to-one schedule.

### 5.1 Upper Bound

As formulated, $P_1$ is a hard allocation problem. As a first step, we will try to get an upper bound. It can be shown [9] that

$$T \;\leq\; \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{a_{ij}}{M} \left[ 1 - (1 - \sigma_i p_{ij})^{\frac{M}{a_{ij}}} \right] \quad (25)$$

Let $x_{ij} = a_{ij}/M$. Variables $x_{ij}$ indicate the percentage of time that station $i$ should transmit to station $j$. Expression (25) implies that the $M x_{ij}$ slots assigned for transmissions from $i$ to $j$ should be equally spaced, separated by a number of $1/x_{ij}$ slots. However, the upper bound cannot be obtained in general for the following reasons: first, $1/x_{ij}$'s may not be integers, and second, even if they are, scheduling all transmissions between all sources and destinations in equally spaced slots may violate constraints (23). If we relax constraints (23), maximizing the upper bound in (25) can be formulated as

$$P_2: \qquad \max_{x_{ij}} \qquad N - \sum_{i=1}^{N}\sum_{j=1}^{N} x_{ij}\,(1-\sigma_i p_{ij})^{1/x_{ij}}$$

$$\text{s. t.} \ \sum_{i=1}^{N} x_{ij} = 1 \ \ \forall\, j\,,\ \sum_{j=1}^{N} x_{ij} = 1 \ \ \forall\, i\,,\ x_{ij} \geq 0 \ \ \forall\, i,j$$

Notice that the solution to $P_2$ is independent of the frame length , $M$. If the values of $x_{ij}$ are known and the number $M$ of slots per frame has been decided upon, we assign of $a_{ij}$ slots to the source-destination pair $(i,j)$, such that constraints (22) are satisfied and

$$\lfloor Mx_{ij} \rfloor \leq a_{ij} \leq \lceil Mx_{ij} \rceil \qquad (26)$$

How the frame length is selected is discussed in detail later.

#### 5.1.1 A Decomposition Heuristic for $P_2$

Unfortunately, problem $P_2$ does not yield an analytical solution. We now develop a heuristic for obtaining the slot assignments. The heuristic is based on the decomposition of $P_2$ into two problems, namely the problems of transmissions to and from a station, discussed in the following paragraphs.

**The Problem of Transmissions *To* a Station:** Here we consider the problem of transmissions to station $j$. We assume that each source has one buffer for storing packets with destination $j$. The probability of a packet arrival at each source $i$ is just $\sigma_i p_{ij}$. This is exactly the single channel problem addressed in [9]. There it was shown that the problem, for destination $j$, can be formulated as,

$$P_3: \qquad \max \qquad 1 - \sum_{i=1}^{N} x_{ij}^{(1)}\,(1-\sigma_i p_{ij})^{1/x_{ij}^{(1)}} \qquad (27)$$

$$\text{subject to} \qquad \sum_{i=1}^{N} x_{ij}^{(1)} = 1 \ \ ,\ \ x_{ij}^{(1)} \geq 0 \quad \forall\, i \qquad (28)$$

and the percentage of time a station should be given permission to transmit to $j$ is

$$x_{ij}^{(1)} \;=\; \frac{\ln(1-\sigma_i p_{ij})}{\sum_{m=1}^{N} ln(1-\sigma_m p_{mj})} \qquad \forall\, j \qquad (29)$$

**The Problem of Transmissions *From* a Station:** We now consider the situation that arises when one source $i$ transmits to many destinations. In a single channel environment the obvious solution is to allow the source to transmit whenever it has a new packet, no matter what the destination is. However, we require that the source transmits to a certain destination only in a fixed number of slots, and our goal is to find the allocation of slots that maximizes the throughput out of station $i$. We assume that station $i$ has a total of $N-1$ buffers, one buffer for storing a packet to each of the destination stations. The probability of a packet arrival at the buffer that stores packets to $j$ is $\sigma_i p_{ij}$.

It turns out that this problem, $P_4$, for all $i$, can be formulated in a similar way as $P_3$, with summations over destinations instead of sources, yielding the following solution.

$$x_{ij}^{(2)} \;=\; \frac{\ln(1-\sigma_i p_{ij})}{\sum_{n=1}^{N} ln(1-\sigma_i p_{in})} \qquad \forall\, i \qquad (30)$$

How we obtain a solution to problem $P_1$ from $x_{ij}^{(1)}$ and $x_{ij}^{(2)}$ will be discussed shortly.

By equating the right hand sides of (29) and (30), we can obtain the conditions under which a solution $x_{ij}^{*}$ to either one of $P_3$ or $P_4$ will also solve $P_2$. One special case is the uniform traffic, i.e., $\sigma_i p_{ij} = \sigma_k p_{kl}\ \forall\, i,j,k,l$. Then, the optimal schedule is a round robin schedule with $M = N-1$, as the one studied in [6].

### 5.2 Construction of One-to-One Schedules

In this section we assume that each source-destination pair $(i,j)$ has been assigned a number of $a_{ij}$ slots for packet transmissions from $i$ to $j$, a frame size of $M$ slots has been selected, and constraints (22) are satisfied. We are interested in finding, for all $i,j$, how the $a_{ij}$ slots should be placed so that the overall throughput is maximized. This problem, $P_5$, whose formulation can be found in [12], is again a hard problem. However, it can be easily shown that if all $a_{ij}$ satisfy (22), then there always exists a one-to-one schedule. Instead of trying to solve $P_5$ we will construct a schedule by considering each channel independently of the others. In a TT-FR system the channels are identified by the receivers. Allocating slots for transmissions to each receiver separately may result in a transmitter being assigned to transmit to two or more receivers in the same slot, or a one-to-many overall schedule. Similarly, if a FT-TR is used, a many-to-one schedule may be produced. The next step will be to use the algorithm shown in Figure 1 to convert this schedule to a one-to-one schedule. The proof of correctness of the algorithm is omitted due to lack of space. It can be found in [12], where it is shown that the worst case complexity of the algorithm is $O(M^2N^2)$.

Our approach to obtain a near-optimal, one-to-one schedule is based on the golden ratio policy in [9], where only frames of lengths equal to the Fibonacci numbers are considered. It can be described by the following steps.

**Heuristic 1**

1. Solve $P_3$, for all $j$, and obtain $x_{ij}^{(1)}$. Solve $P_4$, for all $i$, yielding $x_{ij}^{(2)}$.
2. Select the smallest Fibonacci number $M \geq N-1$. From $x_{ij}^{(1)}$ ($x_{ij}^{(2)}$) obtain the $a_{ij}^{(1)}$ ($a_{ij}^{(2)}$) that satisfy (26) and (22). For all $(i,j)$ set $a_{ij} = \min\{a_{ij}^{(1)}, a_{ij}^{(2)}\}$.
3. If the $a_{ij}$ satisfy (22), go to Step 4. Otherwise, for all $(i,j)$ such that $\sigma_i p_{ij} \neq 0$, add 1 to $a_{ij}$, if doing so does not violate these constraints. Repeat until the constraints are satisfied or until for all $(i,j)$ with $\sigma_i p_{ij} \neq 0$, adding 1 to $a_{ij}$ would violate the constraints. If the latter is true, repeat adding 1 to all other $a_{ij}$ until the

```
for d = 1 to N (* consider one receiver at a time *)
begin
    list ← empty;
    for t = 1 to M (* consider all transmissions to d *)
        if s is such that $\delta^{t}_{sd} = 1$ and it violates $\sum_{j=1}^{d} \delta^{t}_{sj} \le 1$ then
            $\delta^{t}_{sd} \leftarrow 0$; add s to the list;
    while the list is nonempty
    begin
        s ← the first item of the list; violation ← false; l ← d
        repeat
            find a $t_1$ such that $\delta^{t_1}_{sj} = 0 \ \forall j = 1, \ldots, d$
            $\delta^{t_1}_{sl} \leftarrow 1$;
            if there exists a m such that $\delta^{t_1}_{ml} = 1$ then
            begin
                find a $t_2$ such that $\delta^{t_2}_{il} = 0 \ \forall i = 1, \ldots, N$; $\delta^{t_2}_{ml} \leftarrow 1$; $\delta^{t_1}_{ml} \leftarrow 0$;
                if there is a n < d such that $\delta^{t_2}_{mn} = 1$ then
                    violation ← true; s ← m; l ← n;
            end
        until violation = false
    end
end
```

Figure 1: Algorithm for constructing a one-to-one schedule starting with a one-to-many schedule

constraints are satisfied. Notice that in this case, slots are assigned although they will never be used.

4. If the $a_{ij}$ do not satisfy (20) consider the next Fibonacci number and repeat from Step 2. Otherwise, for all channels, use the golden ratio policy [9] to assign slots for transmissions within a channel, producing schedule $S$. $S$ will be one-to-many (many-to-one) for a TT-FR (FT-TR) system.
5. Given $S$, construct a one-to-one schedule $S'$ using the algorithm in Figure 1 (if $S$ is a many-to-one schedule, use a slight variation of the algorithm), and compute its throughput.
6. Repeat Steps 2 through 5 for the next Fibonacci number, up to an upper limit, $M_{max}$. Select the frame length, and the corresponding schedule, that yields the largest throughput.

Each station has to store its part of the schedule; $M_{max}$ is related to the size of the cache memory available at the stations. Also, the golden ratio policy places transmissions within a channel in intervals close to the ones dictated by (25) [9].

# 6 Other Near-Optimal Schedules

Whenever $Q_{ij} = 1 - (1 - \sigma_i p_{ij})^M \ll 1$ for some pairs $(i, j)$, we can take advantage of the low traffic requirements by using one-to-many, many-to-one or many-to-many schedules that may yield better throughput. The idea is that, instead of assigning one slot per pair of stations, it might be better, in terms of throughput, to assign a single slot for transmission from a station, $i$, to a group of stations, provided that the probability of $i$ receiving a packet for any of the stations in the group, during a time period equal to the frame length, is very small. Heuristic 2 describes our approach, and assumes that the frame length is given.

**Heuristic 2**

1. For all $i$, let $G_i = \{j \mid Q_{ij} \le \Delta \ll 1\}$. Partition $G_i$ in disjoint sets $g_{i1}, \ldots, g_{ik}$ such that $\sum_{j \in g_{il}} Q_{ij} < \epsilon < 1$, $l = 1, \ldots, k$.
2. Run Heuristic 1 for the given $M$, modified so that only one slot is assigned for transmissions from $i$ to the stations in group $g_{il}$, $l = 1, \ldots, k$. The resulting schedule is, in general, a many-to-many schedule.
3. Compute the throughput of the schedule obtained at Step 2, by means of the appropriate expressions, and use it if its throughput is better than the throughput of the one-to-one schedule of the same frame length.

Parameter $\Delta$ controls which destinations will not be assigned a slot of their own for transmissions from $i$. Parameter $\epsilon (> \Delta)$ controls the number of stations that are grouped together. The smaller the values of $\Delta$ and $\epsilon$, the smaller the number of stations in a group. By adjusting the values of the two parameters we can keep the probability of collisions and/or destination conflicts within a slot to an acceptable level. Also, by choosing a small $\Delta$ we can have $G_i = \emptyset \ \forall \ i$, in which case a one-to-one schedule will be produced. Even when $G_i$'s are nonempty, the resulting schedule may be a one-to-many, many-to-one, or many-to-many schedule, depending on the actual slot assignment for transmissions to the groups of stations, making Heuristic 2 very general.

# 7 Numerical Results

## 7.1 Optimized One-To-One Schedules

We consider several types of traffic matrices. The matrix of Network 1 is such that the solution of any of problems $P_3$ or $P_4$ will yield the optimal solution to problem $P_2$. Networks

2, 3 and 4 have a disconnected type, ring type and quasi-uniform traffic matrix, respectively. For these traffic matrices we computed the throughput of a round-robin schedule, the throughput of the schedule produced by Heuristic 1, and an upper bound on the throughput of the optimal solution to problem $P_1$. The upper bound was computed by noting that the sum of the objective functions at the optimal solutions to the $N$ problems $P_3$ ($P_4$), one for each destination (source), is an upper bound for the optimal solution to problem $P_2$, and therefore, an upper bound for problem $P_1$. The minimum of the two sums is taken as the upper bound. The value of $M_{max}$ was set to 987.

Due to space limitations, we only show the traffic matrix of Network 3, as well as the one-to-one schedule produced by Heuristic 1 with $M = 21$ for this matrix. The other matrices can be found in [12]. Our results are summarized in Table 1 where we can see that our heuristic produces schedules that are very close to the upper bound. Similar results have been obtained over a very wide range of traffic parameters. Notice also that our schedules represent a significant improvement over the round-robin schedule, especially when the traffic is far from being uniform (Networks 1, 2 and 3).

## 7.2 Optimized Many-To-Many Schedules

Here we consider a 20-station network (Network 5) with the following traffic parameters.

$$\sigma_0 p_{01} = \sigma_0 p_{02} = \sigma_1 p_{10} = \sigma_1 p_{12} = \sigma_2 p_{20} = \sigma_2 p_{21} = 0.49$$

$$\sigma_i p_{ii} = 0 \; i = 0, \ldots, 19 \; , \; \sigma_i p_{ij} \;=\; 10^{-5} \text{ for all other } i, j$$

For this network the upper bound for one-to-one schedules is 2.223 and the round robin throughput is 0.320. In Table 2 we show the throughput of the many-to-many schedules produced by Heuristic 2 for this network for $\Delta = 0.01$ and $\epsilon = 0.2$ and for frame lengths, $M$, equal to the Fibonacci numbers between 21 and 987. We also show the throughput of the one-to-one schedules.

When $\Delta = 0.01$ and $\epsilon = 0.2$, for all values of $M$ shown, we have $Q_{ij} < \Delta \;\; \forall \; i, j$ s. t. $\sigma_i p_{ij} = 10^{-5}$. The groups produced are as follows:

$$g_{i1} = \begin{cases} \{3, \ldots, 19\}, & i = 0, 1, 2 \\ \{0, \ldots, 19\} - \{i\}, & i = 3, \ldots, 19 \end{cases}$$

The throughput of the many-to-many schedules is always better than that of the one-to-one schedules with the same frame length. For small values of $M$, the improvement is significant, as, in a one-to-one schedule, slots assigned to pairs of stations for which $Q_{ij} \ll 1$ are mostly wasted. Heuristic 2 improves this throughput by assigning only one slot for transmissions to a group of stations for which there is low traffic (see Figure 6 for an example).

# 8 Concluding Remarks

We have considered single-hop lightwave networks employing WDM. The stations may have tunable transmitters and/or tunable receivers. Time is slotted and a frequency-time oriented schedule is used to coordinate packet transmissions. We defined all possible types of schedules based on three parameters: the tunability characteristics, the transmission mode, and the policy used by the transmitting and receiving stations. For the round-robin and random policies we analyzed all types of schedules in terms of their throughput, and we derived expressions that are valid for non-uniform traffic.

We also addressed the problem of schedule optimization and derived heuristics to obtain near-optimal schedules. How our results are affected when the number of available wavelengths is less than the number of stations is investigated in a forthcoming paper.

# References


[1] M. H. Ammar and J. W. Wong. On the optimality of cyclic transmission in teletext systems. *IEEE Trans. Commun.*, COM-35(1):68–73, Jan 1987.

[2] C. A. Brackett. Dense wavelength division multiplexing networks: Principles and applications. *IEEE J. Select. Areas Commun.*, 8(6):948–964, Aug 1990.

[3] M.-S. Chen, *et. al.* A media-access protocol for packet-switched wavelength division multiaccess metropolitan area networks. *IEEE J. Select. Areas Commun.*, 8(6):1048–1057, Aug 1990.

[4] R. Chipalkatti, *et. al.* High speed communication protocols for optical star coupler using WDM. In *Proc. IEEE INFOCOM '92*, May 1992.

[5] I. Chlamtac and A. Ganz. Channel allocation protocols in frequency-time controlled high speed networks. *IEEE Trans. Commun.*, 36(4):430–440, Apr 1988.

[6] A. Ganz. End-to-end protocols for WDM star networks. In *IFIP/WG6.1-WG6.4 Workshop on Protocols for High-Speed Networks*, May 1989.

[7] A. Ganz and Z. Koren. WDM passive star - protocols and performance analysis. In *Proc. IEEE INFOCOM '91*, Apr 1991.

[8] P. A. Humblet, *et. al.* An efficient communication protocol for high-speed packet-switched multichannel networks. In *ACM SIGCOM '92*, 1992.

[9] A. Itai and Z. Rosberg. A golden ratio control policy for a multiple-access channel. *IEEE Trans. Autom. Control*, AC-29(8):712–718, Aug 1984.

[10] N. Mehravari. Performance and protocol improvements for very high-speed optical fiber local area networks using a passive star topology. *J. Lightwave Tech.*, 8(4):520–530, Apr 1990.

[11] B. Mukherjee. WDM-Based local lightwave networks Part I: Single-hop systems. *IEEE Network Magazine*, pages 12–27, May 1992.

[12] G. N. Rouskas and M. H. Ammar. Analysis and optimization of transmission schedules for single-hop WDM networks. Tech. Rep. GIT-CC-92/38, Georgia Inst. Tech., 1992.

[13] K. M. Sivalingam, *et. al.* Pre-allocation media access control protocols for multiple access WDM photonic networks. In *ACM SIGCOM '92*, 1992.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0.60 | 0.10 | 0.05 | 0.05 | 0.01 | 0.03 | 0.05 |
| 0.05 | 0 | 0.50 | 0.02 | 0.10 | 0.05 | 0.04 | 0.10 |
| 0.10 | 0.05 | 0 | 0.50 | 0.10 | 0.02 | 0.05 | 0.05 |
| 0.05 | 0.10 | 0.03 | 0 | 0.60 | 0.05 | 0.10 | 0.02 |
| 0.04 | 0.02 | 0.05 | 0.03 | 0 | 0.70 | 0.05 | 0.10 |
| 0.10 | 0.15 | 0.03 | 0.05 | 0.04 | 0 | 0.45 | 0.10 |
| 0.04 | 0.05 | 0.05 | 0.01 | 0.10 | 0.05 | 0 | 0.65 |
| 0.55 | 0.10 | 0.01 | 0.10 | 0.05 | 0.10 | 0.04 | 0 |

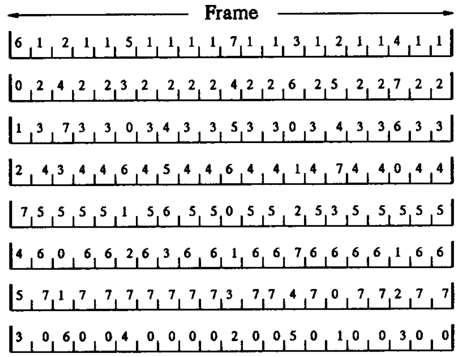


Figure 5: Ring type traffic matrix (Network 3) and one-to-one schedule for $M = 21$

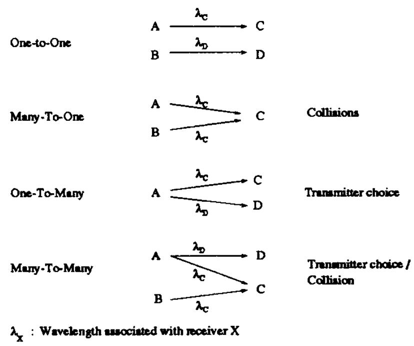


Figure 2: Transmission modes in a TT-FR system

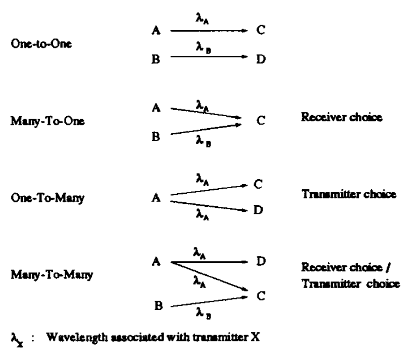


Figure 3: Transmission modes in a FT-TR system

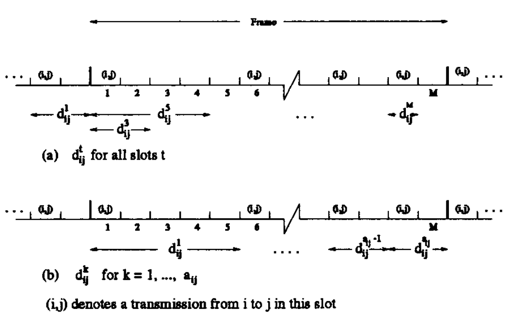


Figure 4: Definition of $d_{ij}^{(t)}$ and $d_{ij}^{(k)}$

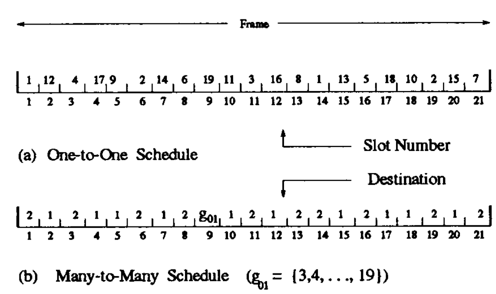


Figure 6: Transmissions *from* station 0 under the one-to-one and many-to-many schedules ($\Delta = 0.01, \epsilon = 0.2$) of frame length $M = 21$ slots, for Network 5

| Net No | One-To-One Throughput RR | One-To-One Throughput Heuristic | Increase from RR | Upper Bound | % from Upper Bound |
|---|---|---|---|---|---|
| 1 | 4.451 | 5.090 | 14.4% | 5.247 | 3.0% |
| 2 | 3.714 | 4.981 | 34.1% | 5.330 | 6.5% |
| 3 | 3.337 | 5.317 | 59.3% | 5.568 | 4.5% |
| 4 | 4.736 | 4.874 | 2.9% | 5.270 | 7.5% |

Table 1: Throughput results and comparisons for 8-station networks using Heuristic 1 (RR = Round-Robin Schedule)

| Frame Length | Throughput One-To-One Schedule | Throughput Many-To-Many Schedule |
|---|---|---|
| 21 | 0.567 | 1.843 |
| 34 | 1.265 | 1.990 |
| 55 | 1.694 | 2.022 |
| 89 | 1.853 | 2.083 |
| 144 | 1.974 | 2.089 |
| 233 | 2.050 | 2.107 |
| 377 | 2.085 | 2.118 |
| 610 | 2.106 | 2.123 |
| 987 | 2.118 | 2.128 |

Table 2: Throughput results for the 20-station FT-TR network 5 using Heuristic 2 ($\Delta = 0.01, \epsilon = 0.2$)