Computing Call Blocking Probabilities in LEO Satellite Networks: The Single Orbit Case

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We study the problem of carrying voice calls over a LEO satellite network, and we present an analytical model for computing call blocking probabilities for a single orbit of a satellite constellation. We have devised a method to solve the corresponding Markov process efficiently for up to 5-satellite orbits. For orbits consisting of a larger number of satellites, we have developed an approximate decomposition algorithm to compute the call blocking probabilities by decomposing the system into smaller sub-systems, and iteratively solving each sub-system in isolation using the exact Markov process. Our approach can capture blocking due to hand-offs for both satellite-fixed and earth-fixed constellations. Numerical results demonstrate that our method is accurate for a wide range of traffic patterns and for orbits with a number of satellites that is representative of commercial satellite systems.

Keywords: LEO Satellite Networks, Performance Analysis, Queueing Theory

1. Introduction

Currently, we are witnessing an increase in the demand for a broad range of wireless telephone and internet services. Satellite based communication is posed to provide mobile telephony and data transmission services on a worldwide basis in a seamless way with terrestrial networks. Satellite systems are location-insensitive, and they can be used to extend the reach of networks and applications to anywhere on the earth.

Satellites can be launched in different orbits, of which, the low earth orbit (LEO), the medium earth orbit (MEO), and the geo-stationary orbit (GEO), are the most well-known. LEO satellites, which are the subject of this work, are placed in orbits at an altitude of less than 2000 Km above the earth. Their orbit period is about 90 minutes, and the radius of the footprint area of a LEO satellite is between 3000 km to 4000 km. The duration of a satellite in LEO orbit over the local horizon of an observer on earth is approximately 20 minutes, and the propagation delay is about 25ms. A few tens of satellites on several orbits are needed to provide global coverage.

In a LEO system, satellites may communicate directly with each other by line of sight using intraplane inter-satellite links (ISL) which connect satellites in the same orbital plane and interplane ISLs which connect satellites in adjacent planes. ISLs introduce flexibility in routing, they can be used to build in redundancy into the network, and they permit two users in different footprints to communicate without the need of a terrestrial system. A constellation of satellites may provide either *satellite-fixed* cell coverage or *earth-fixed* cell coverage. In the first case, the satellite antenna sending the beam is fixed, and as the satellite moves along its orbit, its footprint and the cell move as well. In the case of earth-fixed cell coverage, the earth surface is divided into cells, as in a terrestrial cellular system, and a cell is serviced continuously by the same beam during the entire time that the cell is within the footprint area of the satellite.

As satellites move, fixed and mobile users hand off from one satellite to another (satellite hand-off). The velocity of a satellite is much higher than the velocity of objects on earth. Therefore, the number of hand-offs during a telephone call depends on the call duration, the satellite footprint size and the satellite speed, while the location and mobility of a user only effects the time a hand-off takes place. In an earth-fixed system, all satellite hand-offs occur periodically at the same time. In a satellite-fixed system, a user may be handed off to the satellite behind, as the cell defined by the beam moves away from the user. The newly entered satellite, may or may not have enough bandwidth to carry the handed-off traffic; in the latter case, the call will be dropped. In general, hand-offs in satellite systems impose a big problem from the point of quality of service.

There are several LEO systems currently in operation, such as Argos, VITAsat, ORBCOMM, and Globalstar. These systems differ in many aspects, including the number of orbits and the number of satellites per orbit, the number of beams per satellite, their capacity, the band they operate, and the access method employed. Despite these differences, from the point of view of providing telephony-based services, the principles of operation are very similar, and thus, the analytical techniques to be developed in the proposed work will be applicable to any LEO satellite system that offers such services.

Despite the importance of satellite systems, their performance has not been adequately evaluated. A typical way of modeling a satellite system in the literature is to represent each cell as an M/M/K/K queue. This approach permits the calculation of various useful performance measures, such as the call blocking probability. However, this type of model does not take into account the fact that the amount of traffic in one cell depends on the amount of traffic in one or more other cells. This type of traffic dependencies are taken into account in our models described in Section 2.

In [2], Ganz *et al.*, investigated the distribution of the number of hand-offs and the average call drop probability for LEO satellite systems. Both beam-to-beam and satellite-to-satellite hand-offs were taken into account. Each cell was modeled as an M/M/K/K queue, where K denotes the number of channels per cell, assuming that the number of hand-off calls entering a cell is equal to the number of hand-off calls leaving the cell. In [3], Jamalipour *et al.*, investigated the traffic characteristics of LEO systems and proposed a probability density function to locate the position of each user. In [5], Pennoni and Ferroni described an algorithm to improve the performance of LEO systems by using two queues for each cell, one for new calls and one for hand-off calls. In [7], Ruiz *et al.*, used teletraffic techniques to calculate the blocking and hand-off probabilities. Various channel assignment strategies were investigated. In [1], Dosiere *et al.*, defined a model for calculating the hand-off traffic rate by divideing a street of coverage into small pieces where each piece is equal to the footprint area of a satellite. Uzunalioglu *et al.*, suggested in [8,9] a connection hand-off protocol for LEO satellite systems. Finally, a new traffic load balancing algorithm was proposed by Kim *et al.*, in [4].

In this paper we study the problem of carrying voice calls over a LEO satellite network and we present an analytical model for computing call blocking probabilities for a single orbit of a satellite constellation. The paper is organized as follows. In Section 2 we develop an exact Markov process model under the assumption that satellites are fixed in the sky (i.e., no hand-offs take place), and in Section 3 we present an approximate decomposition algorithm for a large number of satellites. In Section 4 we extend our approach to model hand-offs for both earth-fixed and satellite-fixed coverage. We present numerical results in Section 5, and in Section 6 we conclude the paper by discussing possible directions to which this work may be extended in the future.

2. An Exact Model for the No Hand-Offs Case

Let us first consider the case where the position of the satellites in the single orbit is fixed in the sky, as in the case of geo-stationary satellites. The analysis of such a system is simpler, since no calls are lost due to hand-offs from one satellite to another, as when the satellites move with respect to the users on the earth. This model will be extended in the following section to account for hand-offs in constellations with both earth-fixed and satellite-fixed coverage.

Each up-and-down link of a satellite has capacity to support up to C_{UDL} calls, while each inter-satellite link has capacity equal to C_{ISL} calls. Let us assume that call requests arrive at each satellite according to a Poisson process, and that call holding times are exponentially distributed. We now show how to compute blocking probabilities for the 3 satellites in the single orbit of Figure 1. The analysis can be generalized to

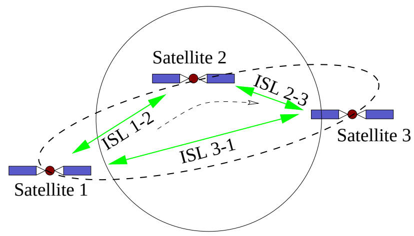

Figure 1. Three satellites in a single orbit

analyze $k > 3$ satellites in a single orbit. For simplicity, we consider only shortest-path routing, although the analysis can be applied to any fixed routing scheme whereby the path taken by a call is fixed and known in advance of the arrival of the call request.

Let n_{ij} be a random variable representing the number of active calls between satellite i and satellite j, $1 \leq i, j \leq 3$, regardless of whether the calls originated at satellite i or j. Let λ_{ij} (respectively, $1/\mu_{ij}$) denote the arrival rate (resp., mean holding time) of calls between satellites i and j. Then, the evolution of the three-satellite system in Figure 1 can be described by the six-dimensional Markov process $\underline{n} = (n_{11}, n_{12}, n_{13}, n_{22}, n_{23}, n_{33})$. Also let $\underline{1}_{ij}$ denote a vector with zeros for all random variables except random variable n_{ij} which is 1. The state transition rates for this Markov process are given by $r(\underline{n}, \underline{n} + \underline{1}_{ij}) = \lambda_{ij} \ \forall \ i, j$, and $r(\underline{n}, \underline{n} - \underline{1}_{ij}) = n_{ij} \ \mu_{ij} \ \forall \ i, j, \ n_{ij} > 0$. The first transition is due to the arrival of a call between satellites i and j, while the second is due to the termination of a call between satellites i and j.

Due to the fact that some of the calls share common up-and-down and inter-satellite links, constraints (1)-(6) are imposed on the state space. Constraint (1) ensures that the number of calls originating (equivalently, terminating) at satellite 1 is at most equal to the capacity of the up-and-down link of that satellite. Note that a call that originates and terminates within the footprint of satellite 1 captures two channels, thus the term $2n_{11}$ in constraint (1). Constraints (2) and (3) are similar to (1), but correspond to satellites 2 and 3, respectively. Finally, constraints (4)-(6) ensure that the number of calls using the link between two satellites is at most equal to the capacity of that link.

$$2n_{11} + n_{12} + n_{13} \leq C_{UDL} \tag{1}$$
$$n_{12} + 2n_{22} + n_{23} \leq C_{UDL} \tag{2}$$
$$n_{13} + n_{23} + 2n_{33} \leq C_{UDL} \tag{3}$$
$$n_{12} \leq C_{ISL} \tag{4}$$
$$n_{13} \leq C_{ISL} \tag{5}$$
$$n_{23} \leq C_{ISL} \tag{6}$$

It is straightforward to verify that the Markov process for the three-satellite system shown in Figure 1 has a closed-form solution which is given by:

$$P(\underline{n}) = P(n_{11}, n_{12}, n_{13}, n_{22}, n_{23}, n_{33}) = \frac{1}{G} \frac{\rho_{11}^{n_{11}}}{n_{11}!} \frac{\rho_{12}^{n_{12}}}{n_{12}!} \frac{\rho_{13}^{n_{13}}}{n_{13}!} \frac{\rho_{22}^{n_{22}}}{n_{22}!} \frac{\rho_{23}^{n_{23}}}{n_{23}!} \frac{\rho_{33}^{n_{33}}}{n_{33}!} \tag{7}$$

where G is the normalizing constant and $\rho_{ij} = \lambda_{ij}/\mu_{ij}, \ i, j = 1, 2, 3$, is the offered load of calls from satellite i to satellite j. As we can see, the solution is the product of six terms of the form $\rho_{ij}^{n_{ij}}/n_{ij}!, \ i, j = 1, 2, 3$, each corresponding to one of the six different types of calls. Therefore, it is easily generalizable to a k-satellite system, $k > 3$.

An alternative way is to regard this Markov process as describing a network of six M/M/K/K queues, one for each type of calls between the three satellites. Since the satellites do not move, there are no hand-offs, and as a consequence customers do not move from one queue to another (we will see in Section 4.2 that hand-offs may be modeled by allowing customers to move between the queues). Now, the probability that there are n customers in an M/M/K/K queue is given by the familiar expression $(\rho^n/n!)/\left(\sum_{k=0}^{K}\rho^k/k!\right)$, and therefore, the probability that there are $(n_{11}, n_{12}, n_{13}, n_{22}, n_{23}, n_{33})$ customers in the six queues is given by (7). Unlike previous studies reported in the literature, our model takes into account the fact that the six M/M/K/K queues are not independent, since the number of customers accepted in each M/M/K/K queue depends on the number of customers in other queues, as described by the constraints (1)-(6).

Of course, the main concern in any product-form solution is the computation of the normalizing constant:

$$G = \sum_{\underline{n}} \frac{\rho_{11}^{n_{11}}}{n_{11}!}\frac{\rho_{12}^{n_{12}}}{n_{12}!}\frac{\rho_{13}^{n_{13}}}{n_{13}!}\frac{\rho_{22}^{n_{22}}}{n_{22}!}\frac{\rho_{23}^{n_{23}}}{n_{23}!}\frac{\rho_{33}^{n_{33}}}{n_{33}!} \tag{8}$$

where the sum is taken over all vectors $\underline{n}$ that satisfy constraints (1) through (6). We now show how to compute the normalizing constant G in an efficient manner. We can write $P(\underline{n})$ as:

$$\begin{aligned} P(n_{11}, n_{12}, n_{13}, n_{22}, n_{23}, n_{33}) &= P(n_{11}, n_{22}, n_{33} \mid n_{12}, n_{13}, n_{23})P(n_{12}, n_{13}, n_{23}) \\ &= P(n_{11} \mid n_{12}, n_{13}, n_{23})\ P(n_{22} \mid n_{12}, n_{13}, n_{23})\ P(n_{33} \mid n_{12}, n_{13}, n_{23})\ P(n_{12}, n_{13}, n_{23}) \\ &= P(n_{11} \mid n_{12}, n_{13})\ P(n_{22} \mid n_{12}, n_{23})\ P(n_{33} \mid n_{13}, n_{23})\ P(n_{12}, n_{13}, n_{23}) \end{aligned} \tag{9}$$

The second step in expression (9) is due to the fact that, once the values of random variables n_{12}, n_{13}, n_{23}, representing the number of calls in each of the inter-satellite links, is fixed, then the random variables n_{11}, n_{22}, and n_{33} are independent of each other (refer also to Figure 1). The third step in (9) is due to the fact that random variable n_{11} depends on n_{12} and n_{13}, and it is independent of the random variable n_{23}; similarly for random variables n_{22} and n_{33}.

When we fix the values of the random variables n_{12} and n_{13}, the number of up-and-down calls in satellite 1 is described by an M/M/K/K loss system, thus $P(n_{11} \mid n_{12}, n_{13}) = \sum_{0 \le 2n_{11} \le C_{UDL} - n_{12} - n_{13}} \frac{\rho_{11}^{n_{11}}}{n_{11}!}$. Similar expressions can be obtained for $P(n_{22} \mid n_{12}, n_{23})$ and $P(n_{33} \mid n_{13}, n_{23})$, corresponding to satellites 2 and 3, respectively. We can now rewrite expression (8) for the normalizing constant as follows:

$$\begin{aligned} G &= \sum_{0 \le n_{12}, n_{13}, n_{23} \le \min\{C_{UDL}, C_{ISL}\}} \frac{\rho_{12}^{n_{12}}\rho_{13}^{n_{13}}\rho_{23}^{n_{23}}}{n_{12}!n_{13}!n_{23}!}\left[\left(\sum_{0 \le 2n_{11} \le C_{UDL} - n_{12} - n_{13}} \frac{\rho_{11}^{n_{11}}}{n_{11}!}\right)\right. \\ &\times \left.\left(\sum_{0 \le 2n_{22} \le C_{UDL} - n_{12} - n_{23}} \frac{\rho_{22}^{n_{22}}}{n_{22}!}\right)\left(\sum_{0 \le 2n_{33} \le C_{UDL} - n_{13} - n_{23}} \frac{\rho_{33}^{n_{33}}}{n_{33}!}\right)\right] \end{aligned} \tag{10}$$

Let $C = \max\{C_{ISL}, C_{UDL}\}$. Using expression (10) we can see that the normalizing constant can be computed in $O(C^3)$ time rather than the $O(C^6)$ time required by a brute force enumeration of all states, a significant improvement in efficiency.

Once the value of the normalizing constant is obtained, we can compute blocking probabilities by summing up all the appropriate blocking states. Consider the 3-satellite orbit of Figure 1. The probability that a call which either originates or terminates at satellite 1 will be blocked on the up-and-down link of that satellite is given by $P_{UDL_1} = \sum_{2n_{11}+n_{12}+n_{13}=C_{UDL}} P(\underline{n})$, while the probability that a call originating at satellite i (or satellite j) and terminating at satellite j (or i) will be blocked by the inter-satellite link (i,j) is $P_{ISL_{ij}} = \begin{cases} 0, & C_{ISL} > C_{UDL} \\ \sum_{n_{ij}=C_{ISL}} P(\underline{n}), & \text{otherwise} \end{cases}$.

Once the blocking probabilities on all up-and-down and inter-satellite links have been obtained using expressions similar to the above, the blocking probability of calls between any two satellites can be easily obtained. We note that the above two expressions explicitly enumerate all relevant blocking states, and thus, they involve summations over appropriate parts of the state space of the Markov process for the satellite orbit. Consequently, direct computation of the link blocking probabilities using these expressions can be computationally expensive. We have been able to express the up-and-down and inter-satellite link blocking probabilities in a way that allows us to compute these probabilities as a byproduct of the computation of the normalizing G. As a result, all blocking probabilities in a satellite orbit can be computed in an amount of time that is equal to the time needed to obtain the normalizing constant, plus a constant. The derivation of the expressions for the link blocking probabilities is a straightforward generalization of the technique employed in (9) and is omitted.

3. A Decomposition Algorithm for the No Hand-Offs Case

Let k be the number of satellites in a single orbit, and N be the number of random variables in the state description of the corresponding Markov process, $N = k(k+1)/2$. Using the method described above, we can compute the normalizing constant G in time $O(C^{N-k})$ as opposed to time $O(C^N)$ needed by a brute force enumeration of all states. Although the *improvement* in the running time provided by our method for computing G increases with k, the value of N will dominate for large values of k. Numerical experiments with the above algorithm indicate that this method is limited to $k = 5$ satellites. That is, it takes an amount of time in the order of a few minutes to compute the normalizing constant G for 5 satellites. Thus, a different method is needed for analyzing realistic constellations of LEO satellites.

In this section we present a method to analyze a single orbit with k satellites, $k > 5$, by decomposing the orbit into sub-systems of 3 or fewer satellites. Each sub-system is analyzed separately, and the results obtained by the sub-systems are combined using an iterative scheme.

In order to explain how the decomposition algorithm works, let us consider the case of a six-satellite orbit, as shown in Figure 2(a). This orbit is divided into two sub-systems. Sub-system 1 consists of satellites 1, 2, and 3, and sub-system 2 consists of satellites 4, 5, and 6. In order to analyze sub-system 1 in isolation, we need to have some information from sub-system 2. Specifically, we need to know the probability that a call originating at a satellite in sub-system 1 and terminating at a satellite in sub-system 2 will be blocked due to lack of capacity in a link in sub-system 2. Also, we need to know the number of calls originating from sub-system 2 and terminating in sub-system 1. Similar information is needed from sub-system 1, in order to analyze sub-system 2.

In view of this, each sub-system is augmented to include two fictitious satellites which represent the aggregate behavior of the other sub-system. In sub-system 1, we add two new satellites, which we call N1 and S1, as shown in Figure 2(b). A call originating at a satellite $i, i = 1, 2, 3$, and terminating at a satellite $j, j = 4, 5, 6$, will be represented by a call from i to one of the fictitious satellites (N1 or S1). Depending upon i and j, this call may be routed differently. For instance, let us assume that $i = 2$ and $j = 4$. Then, in our augmented sub-system 1, this call will be routed to satellite S1 through satellite 3. However, if $j = 6$, the call will be routed to satellite N1 through satellite 1. In other words, satellite N1 (respectively, S1) in the augmented sub-system 1 is the destination for calls of the original orbit that originate from satellite $i, i = 1, 2, 3$ and are routed to satellite $j, j = 4, 5, 6$ in the clockwise (respectively, counter-clockwise) direction in Figure 2(a). Similarly, calls originating from satellite $j, j = 4, 5, 6$, to satellite $i, i = 1, 2, 3$, and are routed in the counter-clockwise (respectively, clockwise) direction, are represented in sub-system 1 as calls originating from N1 (respectively, S1) to i. Again, the originating satellite (N1 or S1) for the call depends on the values of i and j and the path the call follows in the original 6-satellite orbit.

Sub-system 2 is likewise augmented to include two fictitious satellites, N2 and S2 (see Figure 2(b)), which represent the aggregate behavior of sub-system 1. Satellites N2 and S2 become the origin and destination of calls traveling from sub-system 2 to sub-system 1, and vice versa, in a manner similar to N1 and S1 described

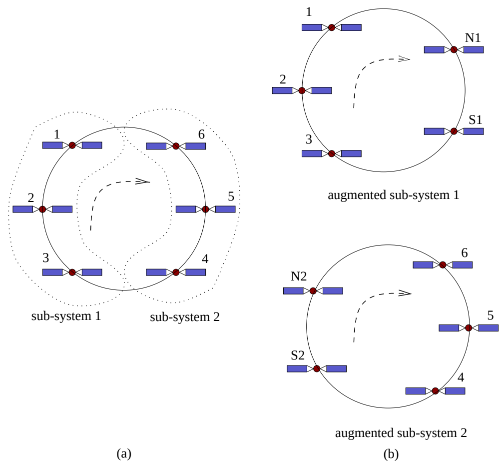

Figure 2. (a) Original 6-satellite orbit, (b) augmented sub-systems

above.

Due to space constraints, a detailed description of our iterative algorithm is omitted, but it can be found in [10]. Below we describe the decomposition algorithm using the 6-satellite orbit shown in Figure 2(a). Recall that $\lambda_{ij}, 1 \leq i \leq j$, is the arrival rate of calls between satellites i and j. For analyzing the augmented sub-systems in Figure 2(b), we will introduce the new arrival rates $\lambda_{i,N1}$, $\lambda_{i,S1}$, $\lambda_{N2,j}$, and $\lambda_{S2,j}$, $i = 1, 2, 3$, $j = 4, 5, 6$. Specifically, $\lambda_{i,N1}$ (respectively, $\lambda_{i,S1}$) accounts for all calls between satellite $i, i = 1, 2, 3$, and a satellite in sub-system 2 that are routed in the clockwise (respectively, counter-clockwise) direction. Similarly, $\lambda_{N2,j}$ (respectively, $\lambda_{S2,j}$) accounts for all calls between sub-system 1 and satellite $j, j = 4, 5, 6$ that are routed in the clockwise (respectively, counter-clockwise) direction.

Initially, we solve sub-system 1 in isolation using:

$$\lambda_{1,N1} = (1 - q_{16})\lambda_{16} + (1 - q_{15})\lambda_{15} \tag{11}$$
$$\lambda_{1,S1} = (1 - q_{14})\lambda_{14} \tag{12}$$
$$\lambda_{2,N1} = (1 - q_{26})\lambda_{26} + (1 - q_{25})\lambda_{25} \tag{13}$$
$$\lambda_{2,S1} = (1 - q_{24})\lambda_{24} \tag{14}$$
$$\lambda_{3,N1} = (1 - q_{36})\lambda_{36} \tag{15}$$
$$\lambda_{3,S1} = (1 - q_{34})\lambda_{34} + (1 - q_{35})\lambda_{35} \tag{16}$$

Quantity $q_{ij}, 1 \leq i \leq 3 < j \leq 6$, represents the current estimate of the probability that a call between a satellite i in sub-system 1 and and satellite j in sub-system 2 will be blocked due to lack of capacity in a link of sub-system 2. For the first iteration, we use $q_{ij} = 0$ for all i and j; how these values are updated in subsequent iterations will be described shortly. Thus, the term $(1 - q_{16})\lambda_{16}$ in (11) represents the *effective* arrival rate of calls between satellites 1 and 6, as seen by sub-system 1; similarly for the other terms in (11)–(16).

The solution to the first sub-system yields an initial value for the probability $p_{ij}, 1 \leq i \leq 3 < j \leq 6$, that a call between a satellite i in sub-system 1 and a satellite j in sub-system 2 will be blocked due to lack of capacity in a link of sub-system 1. Therefore, the effective arrival rates of calls between, say, satellite 1 and satellite 4, that is offered to sub-system 2 can be initially estimated as $(1 - p_{16})\lambda_{16}$. We can now solve sub-system 2 in isolation using [1]:

$$\lambda_{N2,4} = 0 \tag{17}$$
$$\lambda_{S2,4} = (1 - p_{14})\lambda_{14} + (1 - p_{24})\lambda_{24} + (1 - p_{34})\lambda_{34} \tag{18}$$
$$\lambda_{N2,5} = (1 - p_{15})\lambda_{15} + (1 - p_{25})\lambda_{25} \tag{19}$$
$$\lambda_{S2,5} = (1 - p_{35})\lambda_{35} \tag{20}$$
$$\lambda_{N2,6} = (1 - p_{16})\lambda_{16} + (1 - p_{26})\lambda_{26} + (1 - p_{36})\lambda_{36} \tag{21}$$
$$\lambda_{S2,6} = 0 \tag{22}$$

Based on the above discussion, $\lambda_{S2,4}$ in (18) represents the effective arrival rate of calls between a satellite in sub-system 1 and satellite 4, as seen by sub-system 2. Expressions (17)–(22) can be explained in a similar manner. The solution to the second sub-system provides an estimate of the blocking probabilities $q_{ij}, 1 \leq i \leq 3 < j \leq 6$, that calls between satellites in the two sub-systems will be blocked due to lack of capacity in a link of sub-system 2.

The new estimates for q_{ij} are then used in expressions (11) to (16) to update the arrival rates to the two fictitious satellites of augmented sub-system 1. Sub-system 1 is then solved again, and the estimates p_{ij} are updated and used in expressions (17) to (22) to obtain new arrival rates for the fictitious satellites of sub-system 2. This leads to an iterative scheme, where the two sub-systems are solved successively until a convergence criterion (e.g., in terms of the values of the call blocking probabilities) is satisfied.

Orbits consisting of any number $k > 5$ of satellites can be decomposed into a number of sub-systems, each consisting of 3 satellites of the original orbit (the last sub-system may consist of fewer than 3 satellites). The decomposition method is similar to the one above, in that for sub-system l, the remaining satellites are aggregated to two fictitious satellites. Each sub-system is analyzed in succession as described above. The decomposition algorithm described above is similar in spirit to the decomposition algorithms developed for tandem queueing networks with finite capacity queues (see [6]). We note that when employing the decomposition algorithm, the selection of the sub-system size will depend on the number of satellites in the original orbit and how efficiently we can calculate the exact solution of the Markov process associated with each sub-system. It is well known in decomposition algorithms that the larger the individual sub-systems that have to be analyzed in isolation, the better the accuracy of the decomposition algorithm. Thus, as we mentioned above, we have decided to decompose an orbit into sub-systems of the largest size (three of the original satellites plus two fictitious ones) for which we can efficiently analyze the Markov process, plus, possibly, a sub-system of smaller size, if the number of satellites is not a multiple of three.

4. Modeling Hand-Offs

4.1. Earth-Fixed Coverage

Let us now turn to the problem of determining blocking probabilities in a single orbit of satellites with earth-fixed coverage. Let k denote the number of satellites in the orbit. In this case we assume that the earth is divided into k fixed cells (footprints) and that time is divided in intervals of length T such that, during a given interval, each satellite serves a certain cell by continuously redirecting its beams. At the end of each interval, i.e., every T time units, all satellites simultaneously redirect their beams to serve the next footprint along their orbit, and they also hand-off currently served calls to the next satellite in the orbit.

We make the following observations about this system. Hand-off events are periodic with a period of T time units, and hand-offs take place in bulk at the end of each period. Also, there is no call blocking due to hand-offs, since, at each hand-off event a satellite passes its calls to the one following it and simply inherits the calls of the satellite ahead of it. Finally, within each period T, the system can be modeled as one with no

[1] In (17) we have that $\lambda_{N2,4} = 0$ because we assume that calls between satellites in sub-system 1 and satellite 4 are routed in the counter-clockwise direction; similarly for expression (22).

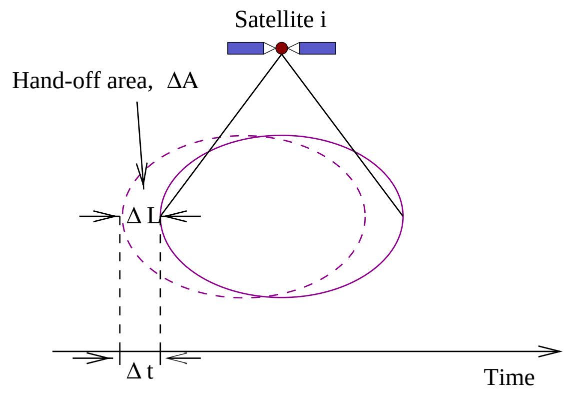

Figure 3. Calculation of the hand-off probability

hand-offs, such as the one described in the previous subsection. Given that the period T is equal to the orbit period (approximately 90 minutes) divided by the number of satellites (e.g., 11 for the Iridium constellation) we can assume that the system reaches steady state within the period, and thus, the initial conditions (i.e., the number of calls inherited by each satellite at the beginning of the period) do not affect its behavior.

Now, since every T units of time, each satellite assumes the traffic carried by the satellite ahead, from the point of view of an observer on the earth, this system appears to be as if the satellites are permanently fixed over their footprints. Hence, we can use the decomposition algorithm presented above to analyze this system.

4.2. Satellite-Fixed Coverage

Consider now satellite-fixed cell coverage. As a satellite moves, its footprint on the earth (the cell served by the satellite) also moves with it. As customers move out of the footprint area of a satellite, their calls are handed off to the satellite following it from behind. In order to model hand-offs in this case, we make the assumption that potential customers are uniformly distributed over the part of the earth served by the satellites in the orbit. This assumption has the following two consequences.

- The arrival rate λ to each satellite remains constant as it moves around the earth. Then, the arrival rate of calls between satellite i and satellite j is given by $\lambda_{ij} = \lambda r_{ij}$, where r_{ij} is the probability that a call originating by a customer served by satellite i is for a customer served by satellite j.

- The active customers served by a satellite can be assumed to be uniformly distributed over the satellite's footprint. As a result, the rate of hand-offs from satellite i to satellite j that is following from behind is proportional to the number of calls at satellite i.

Clearly, the assumption that customers are uniformly distributed (even within an orbit) is an approximation. In Section 6 we will discuss how we are currently extending the results presented in this section to accurately model the situation when customers are not uniformly distributed.

Let A denote the area of a satellite's footprint and v denote a satellite's speed. As a satellite moves around the earth, within a time interval of length Δt, its footprint will move a distance of ΔL, as shown in Figure 3. Calls involving customers located in the part of the original footprint of area ΔA (the hand-off area) that is no longer served by the satellite are handed off to the satellite following it. Let $\Delta A = A\beta\Delta L$, where β depends on the shape of the footprint. Because of the assumption that active customers are uniformly distributed over the satellite's footprint, the probability q that a customer will be handed off to the next satellite along the sky within a time interval of length Δt is $q = \Delta A/A = \beta \Delta L = \beta v \Delta t$. Define $\alpha = \beta v$. Then, when there are n customers served by a satellite, the *rate* of hand-offs to the satellite following it will be αn.

Let us now return to the 3-satellite orbit (see Figure 1) and introduce hand-offs. This system can be described by a continuous-time Markov process with the same number of random variables as the no-hand-offs model of Section 2 (i.e., $n_{11}, \cdots, n_{33}$), the same transition rates as in Section 2, but with a number of additional transition rates to account for hand-offs. We will now derive the transition rates due to hand-offs.

Consider calls between a customer served by satellite 1 and a customer served by satellite 2. There are n_{12} such calls serving $2n_{12}$ customers: n_{12} customers on the footprint of satellite 1 and n_{12} on the footprint of satellite 2. Consider a call between customer A and customer B, served by satellite 1 and 2, respectively. The probability that customer A will be in the hand-off area of satellite 1 but B will not be in the hand-off area of satellite 2 is $q(1-q) = q - q^2$. But we have that $\lim_{\Delta t \to 0} \frac{q^2}{\Delta t} = 0$, so the rate at which these calls experience a hand-off from satellite 1 to satellite 3 that follows it is αn_{12}. Let $\underline{n} = (n_{11}, n_{12}, n_{13}, n_{22}, n_{23}, n_{33})$, and define $\underline{1}_{ij}$ as a vector of zeroes for all variables except variable n_{ij} which is 1. Based on the above discussion, we have $r(\underline{n}, \underline{n} - \underline{1}_{12} + \underline{1}_{23}) = \alpha n_{12},\ n_{12} > 0$. Similarly, the probability that customer B will be in the hand-off area of satellite 2 but A will not be in the hand-off area of satellite 1 is $q(1-q) = q - q^2$. Thus, the rate at which these calls experience a hand-off from satellite 2 to satellite 1 that follows it is again αn_{12}, and $r(\underline{n}, \underline{n} - \underline{1}_{12} + \underline{1}_{11}) = \alpha n_{12},\ n_{12} > 0$. On the other hand, the probability that both customers A and B are in the hand-off area of their respective satellites is q^2, which is $o(\Delta t)$, and thus simultaneous hand-offs are not allowed.

Now consider calls between customers that are both served by the same satellite, say, satellite 1. There are n_{11} such calls serving $2n_{11}$ customers. The probability that exactly one of the customers of a call is in the hand-off area of satellite 1 is $2q(1-q)$, so the rate at which these calls experience hand-offs (involving a single customer) to satellite 3 is $2\alpha n_{11}$; thus, $r(\underline{n}, \underline{n} - \underline{1}_{11} + \underline{1}_{13}) = 2\alpha n_{11},\ n_{11} > 0$. As before, the probability that both customers of the call are in the hand-off area of satellite 1 is q^2, and again, no simultaneous hand-offs are allowed. The transition rates involving the other four random variables in the state description $\underline{n}$ are omitted due to space constraints, but can be derived using similar arguments.

From the queueing point of view, this system is the queueing network of M/M/K/K queues described in Section 2, where customers are allowed to move between queues according to the above transition rates. (Recall that in the queueing model of Section 2, customers are not allowed to move from node to node.) This queueing network has a product-form solution similar to (7). Let γ_{ij} denote the total arrival rate of calls between satellites i and j, including at a rate of λ_{ij}) and hand-off calls (arriving at an appropriate rate). The values of γ_{ij} can be obtained by solving the traffic equations for the queueing network. Let also $\nu_{ij} n_{ij}$ be the departure rate when there are n_{ij} of these calls, including call termination (at a rate of $\mu_{ij} n_{ij}$) and call hand-off (at a rate of $2\alpha n_{ij}$). Also, define $\rho'_{ij} = \gamma_{ij}/\nu_{ij}$. Then, the solution for this queueing network is given by an expression which is identical to (7) except that ρ_{ij} is replaced by ρ'_{ij}. This product-form solution can be generalized in a straightforward manner for any k-satellite orbit, $k > 3$. We can thus use the techniques developed in Section 2 to solve the system involving hand-offs exactly, or we can use the decomposition algorithm presented in Section 3 to solve orbits with a large number of satellites.

5. Numerical Results

In this section we validate the decomposition algorithm by comparing to simulation results. Results using the exact model are omitted due to space constraints, but can be found in [10]. We consider a single orbit of a satellite constellation consisting of 12 satellites, a number representative of typical commercial satellite systems. In the figures presented, simulation results are plotted along with 95% confidence intervals estimated by the method of replications. The number of replications is 30, with each simulation run lasting until each type of call has at least 15,000 arrivals. For the approximate results, the iterative decomposition algorithm terminates when all call blocking probability values have converged within 10^{-6}. In all cases

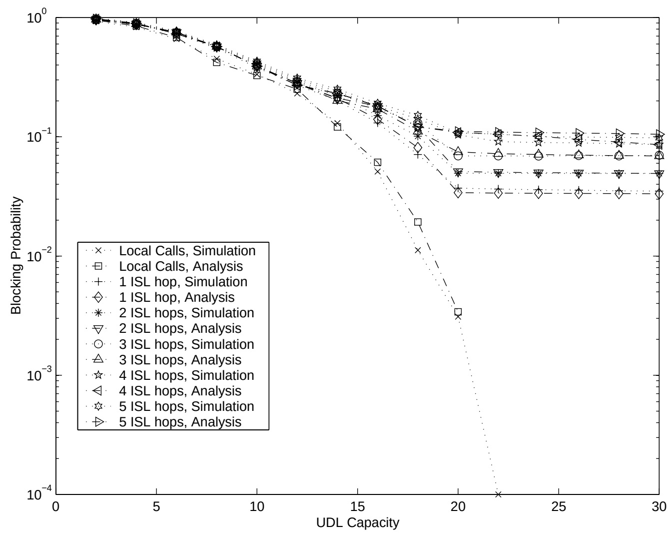

Figure 4. 12-satellite orbit, $\lambda = 5$, $C_{ISL} = 20$, uniform pattern

studied, we have found that the algorithms converges in only a few (less than ten) iterations, taking a few minutes to terminate. On the other hand, simulation of 12-satellite orbits is quite expensive in terms of computation time, taking several hours to complete. We have considered three different traffic patterns: a uniform traffic pattern, a locality pattern were most calls originating at a satellite i are to users in satellites $i-1$, i, and $i+1$, and a two-community pattern.

Figure 4 plots the blocking probability against the capacity C_{UDL} of up-and-down links, when the arrival rate $\lambda = 5$ and the capacity of inter-satellite links $C_{ISL} = 20$, for the uniform traffic pattern. Six sets of calls are shown, one for local calls, and five for non-local calls. Each set consists of two plots, one corresponding to blocking probability values obtained by running the decomposition algorithm of Section 3, and one corresponding to simulation results. Each non-local call for which results are shown travels over a different number of inter-satellite links, from one to five. Thus, the results in Figure 4 represent calls between all the different sub-systems in which the 12-satellite orbit is decomposed by the decomposition algorithm.

From the figure we observe the excellent agreement between the analytical results and simulation. The behavior of the curves can be explained by noting that, when the capacity C_{UDL} of up-and-down links is less than 20, these links represent a bottleneck. Thus, increasing the up-and-down link capacity results in a significant drop in the blocking probability for all calls. When $C_{UDL} > 20$, however, the inter-satellite links become the bottleneck, and non-local calls do not benefit from further increases in the up-and-down link capacity. We also observe that, the larger the number of inter-satellite links over which a non-local call must travel, the higher its blocking probability, as expected. The blocking probability of local calls, on the other hand, drops to zero for $C_{UDL} > 20$ since they do not have to compete for inter-satellite links.

Figures 5 and 6 are similar to Figure 4 but show results for the locality and 2-community traffic patterns, respectively. For the results presented we used $\lambda = 5$ and $C_{ISL} = 10$, and we varied the value of C_{UDL}. We observe that the values of the call blocking probabilities depend on the actual traffic pattern, but the behavior of the various curves is similar to that in Figure 4. Finally, in Figure 7, we fix the value of C_{UDL} to 20, and we plot the call blocking probabilities for the 2-community traffic pattern against the capacity C_{ISL} of the inter-satellite links. Overall, the results in Figures 4–7 indicate that analytical results are in good agreement with simulation over a wide range of traffic patterns and system parameters.

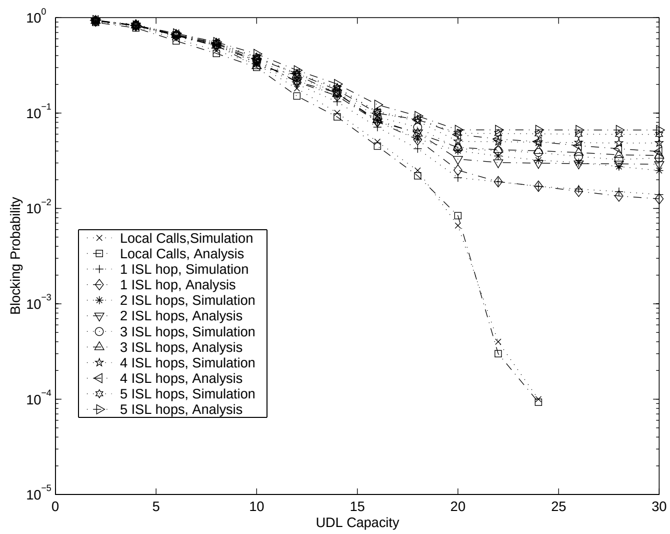

Figure 5. 12-satellite orbit, $\lambda = 5$, $C_{ISL} = 10$, locality pattern

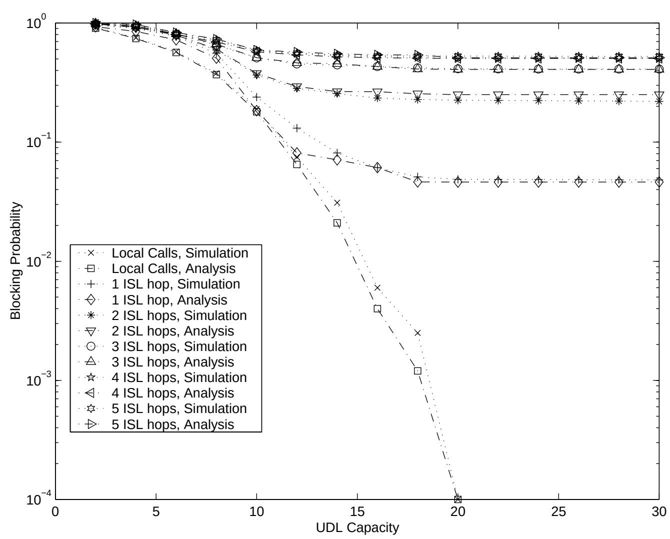

Figure 6. 12-satellite orbit, $\lambda = 5$, $C_{ISL} = 10$, 2-community pattern

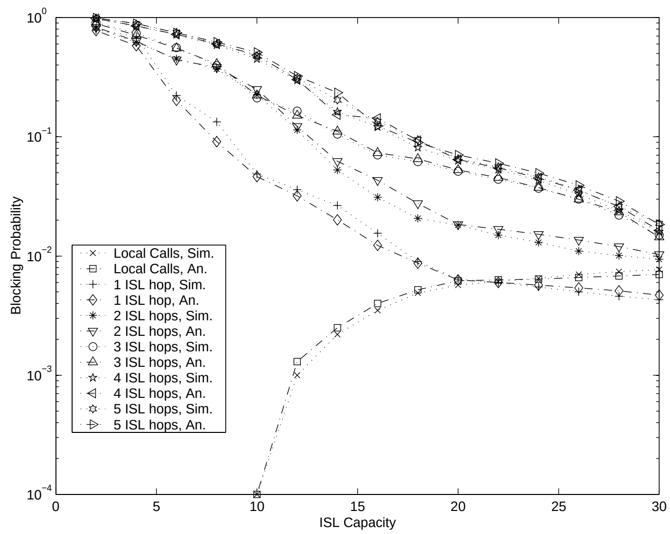

Figure 7. 12-satellite orbit, $\lambda = 5$, $C_{UDL} = 20$, 2-community pattern

6. Concluding Remarks

We have presented an analytical model for computing blocking probabilities, for new and hand-off calls, for a single orbit of a LEO satellite constellation. We have devised a method for solving the exact Markov process efficiently for up to 5-satellite orbits. For orbits consisting of a larger number of satellites, we have developed an approximate decomposition algorithm to compute the call blocking probabilities by decomposing the system into smaller sub-systems, and iteratively solving each sub-system in isolation using the exact Markov process. We have also shown how our approach can capture blocking due to hand-offs for both satellite-fixed and earth-fixed orbits.

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