RWA in WDM Rings: Efficient Exact Formulations Based on Maximal Independent Sets

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Outline

- Routing and Wavelength Assignment (RWA)
- Existing ILP Formulations
- New ILP Formulations Based on
  - MIS Decomposition
  - MIS Selection
- Numerical Results
- Conclusion and Future Research Directions
Why “RWA in Rings”? 

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- subproblem of all optical network design problems
  → speed up “what-if” analysis to test sensitivity of solution to forecast demands, cost projections, price structures, etc.
- intellectually appealing!
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- Why “Rings”?
  - ring topologies prevalent today and in foreseeable future
  - insight into RWA problem in mesh topologies
Routing and Wavelength Assignment (RWA)

- Fundamental control problem in optical networks

- Objective: for each connection request determine a lightpath, i.e.,
  - a path through the network, and
  - a wavelength

- Two variants:
  1. **online RWA**: connection requests arrive/depart dynamically
  2. **static RWA**: a set of traffic demands to be established simultaneously
Static RWA

Input:
- network topology graph $G = (V, E)$
- traffic demand matrix $T = [t_{sd}]$

Objective:
- minRWA: establish all demands with the minimum # of $\lambda$s
- maxRWA: maximize established demands for a given # of $\lambda$s

Constraints:
- wavelength continuity: each lightpath uses the same $\lambda$ along path
- distinct wavelength: lightpaths using the same link assigned distinct $\lambda$s

NP-hard problem (both variants)
Solution Approaches

1. ILP formulations
   - Link-based
   - Path-based
   - MIS-based

2. Heuristics
   - Decomposition: R & WA
   - Multi-layer graph
   . . .
Challenges

- Existing approaches do not scale well with:
  - network size
  - number of wavelengths
- Quality of heuristics is difficult to characterize
- Large $\lambda$ regime not explored
RWA: Symmetry
Link ILP Formulation

- Nodes/links are entities of interest
- Focus on traffic demand to and from nodes, on links
- Bridging variable: demand between nodes on links
Nodes/paths are entities of interest
Demand is still between nodes
For each given demand node pair, list all paths
   → typically, a subset of all paths

assign variable to path traffic flow → implicitly identifies demand
for each link, sum up path flow variables
   → constrain with capacities
RWA As Graph Coloring

[Diagram showing a graph with nodes and edges labeled 1 to 6 on the left, and a smaller graph on the right with nodes 1 to 6.]
Independent set: a set of vertices in a graph no two of which are adjacent

Maximal independent set: not a subset of any other independent set
Precompute $k$ paths for each source-destination pair

Create the path graph $G_p$:
- each node in $G_p$ corresponds to a path in the original network
- two nodes connected in $G_p$ if corresponding paths share a link

Enumerate the MISs of $G_p$

Set up ILP to assign wavelengths to each MIS
## Comparison

<table>
<thead>
<tr>
<th>Formulation</th>
<th># Variables</th>
<th># Constraints</th>
<th>Symmetry?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link</td>
<td>$O(N^4W)$</td>
<td>$O(N^3W)$</td>
<td>Yes</td>
</tr>
<tr>
<td>Path</td>
<td>$O(N^2W)$</td>
<td>$O(N^2W)$</td>
<td>Yes</td>
</tr>
<tr>
<td>MIS</td>
<td>$O(3^{N^2/3})$</td>
<td>$O(N^2)$</td>
<td>No → future-proof</td>
</tr>
</tbody>
</table>
Running Time Results, $W = 120$
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Running Time Results, $W = 120$
Clockwise paths do not intersect with counter-clockwise paths:

\[ G_p = G_{cw}^p \cup G_{ccw}^p \]

\( M, M_{cw}, M_{ccw} \): # of MISs of \( G_p, G_{cw}^p, G_{ccw}^p \):

\[ M_{cw} = M_{ccw} = \sqrt{M} \]

→ orders of magnitude decrease in # of variables/size of formulation

Slight modifications to formulation
Further Decomposition: MISD-4

- Consider **clockwise** direction only
  - similar steps for counter-clockwise

- Partition ring in two parts such that:

\[ G_{cw}^p = G_{cw,0}^p \cup G_{cw,1}^p \cup G_{cw,core}^p \]
Express each MIS $m$ of $G^{cw}_p$ as:

$$m = m^0 \cup m^1 \cup q$$

Modify the formulation appropriately

- # MIS variables ↓
- # constraints ↑

Recursively partition the two ring parts to effect higher-order decompositions (MISD-8, MISD-16, . . .)
Results: # of MIS Variables

![Graph showing the number of MIS variables for different values of N.]

- MIS
- MISD−2
- MISD−4
- MISD−8
Running Time Results, $W = 120$
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Running Time Results, $W = 120$
Results: Scalability with $W$

![Graph showing scalability with $W$]
16-node ring solution takes $< 1$ sec for any # of $\lambda$s
→ problem solved!
Discussion

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- Can we apply MIS decomposition to mesh networks?
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  \(\rightarrow\) problem solved!

- Can we apply MIS decomposition to mesh networks?
  - yes – and it works well
  - but: size of initial MIS set orders of magnitude larger
    \(\rightarrow\) back to the drawing board
# of MIS Variables

![Graph showing the number of MIS Variables with a log-log scale. The graph compares different MIS types: MIS, MISD-2, MISD-4, and MISD-8. The x-axis represents the value of N, and the y-axis represents the number of MISs. Each line represents a different MIS type, with distinct markers and colors.](image)
Can We Do Better?

**Graph Description:**
- **X-axis:** Number of links (N)
- **Y-axis:** SOL Time (s)
- **Legend:**
  - link
  - path
  - MIS
  - MISD−2
  - MISD−4
  - MISD−8
  - Weighted MIS
- Plot shows the increase in SOL time with the number of links for different link weight configurations.

**Key Findings:**
- The SOL time increases exponentially with the number of links.
- The Weighted MIS configuration has the highest SOL time, followed by MISD−8, MISD−4, MISD−2, path, and link.
- The SOL time for the Weighted MIS is significantly higher than for other configurations.

**Graph Notes:**
- **Mem:** 7200
- **tLim:** 0.001
- The graph indicates that increasing the number of links significantly affects the SOL time, suggesting a need for more efficient link weight configurations.
Observations

- # of MIS variables: millions or more
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- Many disjoint optimal solution sets exist
  → Some MIS variables important, others not
- Can we identify the important ones?
MIS Selection

- Prune useless MIS variables
  → those containing paths with no traffic
- Rank remaining MIS variables in decreasing order of weight:
  - path (node) weight:
    \[ w = \text{degree}^2 \times \text{traffic} \]
  - MIS weight:
    \[ \sum_{\text{node } i \in \text{MIS}} w_i \]
- Include only top 10% of ordered MIS variables in formulation
Results

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MIS Generation

- Large rings and mesh networks:
  - bottleneck shifts from CPLEX to enumeration of MIS variables
  - MIS set cannot fit in memory
- New algorithms needed: enumerate only most promising MIS variables
  - topic of ongoing research
RWA problem can be solved efficiently in rings
→ extensive “what-if” analysis now possible

Current research focuses on:
- extending MIS selection to mesh networks
- efficient ILP formulations for optical network design problems
  - incorporate MIS decomposition for RWA
  - employ problem-specific knowledge